R E P O R T R E S U M E S

ED 002 960 24

PROBLEM SOLVING AND PRIOR LEARNING.

BY- SCANDURA, JOSEPH M.
FLORIDA ST. UNIV., TALLAHASEE

REPORT NUMBER CRP-S-097 PUB DATE 65
STATE UNIV. OF N.Y., ALBANY, RESEARCH FOUNDATION
GRANT OEG-410209
EDRS PRICE MF-$0.18 HC-$3.04 76P. JOURNAL OF EXPERIMENTAL EDUCATION, 33(2) 1964

DESCRIPTORS- *PROBLEM SOLVING, *LEARNING PROCESSES, *ACADEMIC ABILITY, *INDIVIDUAL INSTRUCTION, PACING, RETENTION, TRANSFER OF TRAINING, TASK PERFORMANCE, MATHEMATICS, SCIENCES, COLLEGE STUDENTS, BASIC SKILLS, TALLAHASSEE, FLORIDA, ALBANY, BUFFALO, NEW YORK

VARIOUS WAYS WERE EXPLORED BY WHICH PRIOR OR PREREQUISITE LEARNING AFFECTS THE LEARNING OF HIGHER ORDER (CRITERION) MATERIAL WHICH BUILDS ON PREVIOUS EDUCATIONAL FOUNDATIONS. THE INVESTIGATION WAS CONCERNED WITH THREE INSTRUCTIONAL VARIABLES (PACING, PRACTICE, AND NUMBER OF STAGES IN A LEARNING HIERARCHY) AND THEIR EFFECTS ON LEARNING RETENTION AND TRANSFER OF PROBLEM-SOLVING SKILLS. IN ADDITION, RELATIONSHIPS BETWEEN THESE INPUT VARIABLES WERE EXPLORED. EXPERIMENTS USED IN THE PROJECT WERE ENTITLED AS FOLLOWS--(1) "PROBLEM SOLVING AND PRIOR LEARNING," (2) "ALGORITHM LEARNING AND PROBLEM SOLVING," (3) "PRIOR LEARNING, PRESENTATION ORDER, AND PREREQUISITE PRACTICE IN PROBLEM SOLVING," AND (4) "PREREQUISITE PRACTICE AND CRITERION FORM IN MATHEMATICS LEARNING." THESE EXPERIMENTS WERE RUN ON GROUPS OF 15 TO 40 COLLEGE STUDENTS. THESE SUBJECTS WERE TESTED ON CRITERION, GENERALIZATION, AND NONSPECIFIC TRANSFER TASKS IMMEDIATELY AFTER LEARNING AND, AGAIN, AFTER 5 WEEKS (USING PRACTICE WITH SUBORDINATE MATERIAL AND CRITERION PRACTICE). STANDARD ANALYSIS VARIANCE PROCEDURES PROVIDED THE BASIS FOR MOST OF THE INFERENCES. LEARNING ABILITY WAS FOUND TO BE FAR FROM INNATE. IT DEPENDED FUNDAMENTALLY ON THE AMOUNT OF PREREQUISITE LEARNING ALREADY AVAILABLE. THE NEED FOR FURTHER RESEARCH IN THE FIELD OF INDIVIDUALIZED INSTRUCTION IS STRESSED. (JH)

ED002960

SPECIAL
DEPARTMENT OF
HEALTH, EDUCATION, AND WELFARE
HEW 94
GPO 861—0-61/333

PROBLEM SOLVING AND PRIOR LEARNING

Project No. S-097 HER-e

Joseph M. Scandura
Florida State University
Tallahassee, Florida

May 1, 1964 to January 31, 1966

U. S. DEPARTMENT OF HEALTH, EDUCATION AND WELFARE
Office of Education
This document has been reproduced exactly as received from the person or organization originating it. Points of view or opinions stated do not necessarily represent official Office of Education position or policy.

PROBLEM SOLVING AND PRIOR LEARNING

Cooperative Research Project No. S-097

Author

JOSEPH M. SCANDURA

The Florida State University on behalf of the Research Foundation of State University of New York

1965

The research reported herein was supported by the Cooperative Research Program of the Office of Education, U. S. Department of Health, Education and Welfare.

PREFACE

Four distinct, though highly related, experiments are reported herein. The first deals with the effects of practice at prerequisite and criterion levels and is entitled, "Problem Solving and Prior Learning." The second deals with relationships between prerequisites and problem solving algorithms and is entitled, "Algorithm Learning and Problem Solving." The third is the major study in the series and deals with the effects and interactions of prior learning, presentation order, and practice on problem solving. It is entitled, "Prior Learning, Presentation Order, and Prerequisite Practice in Problem Solving." The fourth is a pilot study, using mathematical materials, and is entitled, "Supplement: Prerequisite Practice and Criterion Form in Mathematics Learning." The fourth study was conducted and is reported as an addition to the work outlined in the original contract.

The first two reports have already been accepted for publication by the Journal of Experimental Education. A tentative commitment, pending careful review, has also been obtained regarding the other two reports.

Originally, contract S-097 was negotiated between the U. S. Office of Education and the Research Foundation of the State University of New York. After the principle investigation moved to The Florida State University, a subcontract was negotiated between the Research Foundation and that university to insure completion of the project.

The writer gratefully acknowledges the space and other facilities made available by the State University of New York at Buffalo and The Florida State University for conducting this research. The writer also gratefully acknowledges the assistance of Richard DeMunda, Gabriel DeMunda, Robert Magee, Merlyn Behr, Richard Eiswirth, and Judith Anderson. Without their

help and the participation of over 400 high school and college students, this research would not have been possible.

J.M.S.

P.S. All four reports have been accepted for publication in the Journal of Experimental Education. They will appear simultaneously, some time during 1966, under the titles:

(1) Problem Solving and Prior Learning,

(2) Algorithm Learning and Problem Solving,

(3) Prior Learning, Presentation Order, and Prerequisite Practice in Problem Solving, and

(4) Supplement: Prerequisite Practice and Criterion Form in Mathematics Learning.

J. exp. Educ., in pr

Problem Solving and Prior Learning

Joseph M. Scandura

Florida State University

Abstract

Eighty Ss were presented with subordinate material. Then, a criterion task was defined and illustrated. Two variables were considered: practice with the subordinate material (P) and criterion practice (C). The Ss were tested on criterion (R), generalization (RG), and non-specific transfer (N) tasks immediately after learning and after five weeks. P affected R and, to a lesser extent, RG performance on the immediate tests, but not after five weeks. There were no significant effects due to C. The effects of P were attributed to increasing the degree of subordinate learning which, in turn, facilitated criterion learning.

Problem Solving and Prior Learning[1]

Joseph M. Scandura

Florida State University

This study was concerned with how learning to solve problems depends on prior learning. In exploring a related problem, Scandura (1964a, 1964b) found evidence which suggests that mere presentation of subordinate material is not always sufficient to insure subsequent learning when the subordinate material is used to describe the higher order material. The implications of this observation, if borne out experimentally, could be substantial. Thus, rapidly covering preliminary material, so as to get to the main topic more quickly, may in the long run defeat its own purpose.

Of course, good teachers have long recognized that when material is introduced determines to a large extent how well it is learned. Nonetheless, to the author's knowledge, there are no published experimental studies which bear directly on the present problem. This does not mean that related questions have gone unnoticed. Kersh (1955) reviewed some earlier studies (Brownell, 1931; White, 1934), which were concerned with the effects of familiar and unfamiliar settings on problem solving in arithmetic. These studies were, in effect, concerned with relationships between superordinate material (the problems) and prerequisites, but the results were necessarily in the form of R-R laws. No attempt was made to manipulate familiarity. The recent work of Gagné et al. (1961a, 1961b, 1962a, 1962b) and Ausubel et al. (1960, 1962, 1963) was also related to this research in that their concern was with the relationships between subordinate and superordinate learning.

---

[1]This research was supported by the Cooperative Research Program of the U.S. Office of Education, Project S-097. Thanks are due Richard DeMunda who was responsible for obtaining the subjects and assisting with both the conduct of the experiment and scoring the materials.

Gagné, however, was concerned with analyzing complex cognitive skills into component skills which were more readily taught and not with the presentation of continuous discourse in terms of subordinate associations, concepts, and principles. Ausubel dealt with the effects of "advance organizers," which consist of general introductory material, on later learning, but not with the situation described.

The purpose of this experiment was to determine the effects on the learning, transfer, and retention of problem solving skills of practice at both the prerequisite and criterion levels in a learning hierarchy.

METHOD

Materials, apparatus, and subjects.- The materials used in this experiment were based on some abstract materials (Scandura, 1964a) developed for use in problem solving research. This material was based on an extension of card tasks frequently used to study concept formation (e.g., Bruner, Goodnow, and Austin, 1956). Sixty-four cards were used. Each card had two objects on it, six two-valued attributes in all. Corresponding to each of the twelve attribute values was a common symbol. Cards were assigned one of several possible labels. A symbol set was said to be associated with a given labeled card if, and only if, the symbols in the set were related to the card properties in a specified way. A card problem consisted of determining all symbol sets associated with each card comprising the problem. There was an algorithm for solving each of the many possible problem types. Problem types differed in the card labels used and/or the algorithm pattern and number of solution sets.

For the purposes of the present experiment, the material was divided into three levels. The level one material was used to describe the level two and

three material and the level two material was used to describe the level three material. At level one were the twelve one-to-one correspondences between the card properties and the symbols which represented those properties. There was a common principle relating each property to the corresponding symbol. For example, S_1 and T_2 stood for the first object is Small and the second is a Triangle, respectively. Level two consisted of labeling the cards (either +all or +1) and defining (criterion) principles by which it was possible to decide, for each set of symbols, whether it was or was not associated with any given labeled card. A set of symbols was associated with a +all (+1) card if, and only if, all (exactly one) of the symbols corresponded to the card properties. Level three consisted of defining the problems (i.e., finding all symbol sets associated with each of three labeled cards) and pointing out some of the relationships between an illustrative problem (see R problem below) and its solution. Every symbol used in writing the solution sets corresponded to some property of each of the +all cards and the symbol which corresponded to the +1 card property appeared in each solution set. In no case was information given as to how the solution was obtained. The level one and level two material, taken collectively, was referred to as the prerequisite material. The criterion material consisted of level three.

Three types of problems were used. The routine (R) problems consisted of two +all cards and a +1 card, had θ - - - as an algorithm pattern[2] (with eight solution sets), and were used as practice problems for level three (with knowledge of results) and to test for problem-solving ability. The

---

[2]Dashes (-) represent symbols used in solution sets and encircled dashes (θ) represent those symbols that correspond to +1 or +2 card properties. For more details see the article by Scandura (1964a).

generalization (RG) problems also consisted of two +all cards and a +1 card, but had $\theta \overset{1}{-} \theta -$ and $\theta \overset{1}{\theta} \theta -$ as algorithm patterns[2] (with 8 and 6 solution sets), and were used to measure transfer. The novel (N) problems each consisted of three cards (+all, -all, and +2) with algorithm patterns[2] - $\theta \overset{2}{\theta} \theta$ and $\theta \overset{2}{\theta} \theta \theta$ and 6 and 6 solution sets, respectively. The N problems were used to test for less specific transfer.

The materials were presented in booklet form and consisted of a one-half to one-and-a-half page double-spaced introduction to each level of the material and a number of practice problems (a sufficient number to keep even the fast Ss busy throughout the corresponding practice period) with answers on the following page. In addition, a number of routine arithmetical and algebraic factoring problems (filler) were constructed. All materials were presented on dittoed sheets to 80 eleventh grade college preparatory mathematics Ss in the Niagara Falls, New York, school system. None of these Ss was in an honors section.

Design.- Two variables were independently manipulated: prerequisite practice (P) and criterion practice (C). P was defined in terms of the amount of prerequisite practice. Half of the 80 Ss were given a total of 11 minutes of prerequisite practice, including an opportunity to check their answers; the others were given none. The criterion practice variable refers to the time allotted for attacking the R problems. Half of the Ss, at each prerequisite practice level (NP-C, P-C), were given 11 minutes of R-problem practice (with answers); the others were given none (NP-NC, P-NC). In sum, the 80 Ss were assigned randomly, subject to equal cells, to four treatment groups in a 2 x 2 factorial design.

Procedure.- The Ss were run in five sections of sixteen (four in each treatment group). The time allotted for each part of the instructional period was as follows: level one material, four minutes; level one practice, four minutes; level two-A (+all criterion) material, four minutes; level two-A practice, four minutes; level two-B (+1 criterion) material, three minutes; level two-B practice, three minutes; level three material, six minutes; and level three practice, eleven minutes. The no practice Ss (whether prerequisite, criterion, or both) were given irrelevant filler problems to work (and check) at the appropriate level(s). Not only were identical materials presented to all groups, but the materials were equally spaced. Provision was made for each S to check his answers to all questions and problems by turning to the next page in his booklet.

Immediately after the instruction and approximately five weeks later, the Ss were given three two-problem tests in the order, R-RG-N.[3] The experimenter and four assistants recorded the time taken by each S to complete each test. A maximum of ten minutes was allowed for each. The immediate and retention problems had identical algorithm patterns and card types, but the cards, themselves, differed. All Ss were allowed four minutes to study the -all and +2 criteria just prior to the N testing.

The original instruction-test period took about 85 minutes while the retention testing took about 35 minutes.

---

[3]Reliability estimates of .89, .68, and .66 for the R, RG, and N tests, respectively, were obtained by computing the correlation coefficients between the correct scores on the two problems in each test. The problems were randomly designated "X" or "Y" so that half of the X(Y) scores were from problem one and half from problem two.

RESULTS

The number of correct solution set scores varied from 0 to 16 (100%). A summary of the means and standard deviations is given in Table 1. An analysis of variance revealed that R performance immediately after learning

---

Insert Table 1 about here

---

was an increasing function of prerequisite practice ($F=4.71$, $df=1/76$, $p<.05$). The effect due to criterion practice was not significant ($F=2.12$, $df=1/76$, $p>.10$). There were 8, 7, 4, and 2 Ss (of 20) in the NP-NC, NP-C, P-NC, and P-C groups, respectively, who had zero correct scores. A significantly greater proportion of those Ss who had had prerequisite practice had nonzero correct scores ($\chi^2=5.23$, $df=1$, $p<.025$). Further comparison of the NP-NC and NP-C score distributions suggested that C practice may have merely increased the proficiency level of those Ss who would have achieved some degree of success without such practice. The generalization (RG) effect due to P just did attain an acceptable level of significance ($F=3.97$, $df=1/76$, $p=.05$) but there was no C effect ($F<1$). An identical analysis on the N (correct) scores revealed no significant treatment effects. Only the P by C interaction resulted in an F greater than one ($F=2.83$, $df=1/76$, $.05<p<.10$). The P-C Ss performed somewhat better than the others.

Analyses of variance on the retention (correct) scores indicated that all treatment effects had disappeared after five weeks. None of the F-ratios approached significance. Repeated measures analyses, including both immediate and retention scores, were also conducted and are shown in Table 2. These analyses indicated a high degree of forgetting after five weeks on the R, RG,

---

Insert Table 2 about here

---

and N tests ($F=47.57$, 41.29, and 29.91, $df=1/72$, $p<.001$). A significant interaction between prerequisite practice and retention on the R scores ($F=8.88$, $df=1/72$, $p<.01$) substantiated the results of the separate analyses. Prerequisite practice facilitated R-performance immediately after learning, but not after five weeks. In order to simplify these analyses, all frequencies were equalized at 19 by discarding (randomly) the data from one S in the NP-C and P-NC groups. Since overall performance on the retention tests was extremely poor and probably reflected near minimal performance, the results of these repeated measures, as well as simple retention, analyses must be viewed with caution. The lack of treatment effects after five weeks may merely have been due to relatively lower scale sensitivity to the treatments. Such differential sensitivity could have contributed to both the retention by P interaction (repeated measures analysis) and the P main effect (retention analysis) on the R scores.

Fewer solution sets (including incorrect sets) were offered on the retention tests. A χ^2 test indicated that a greater percentage of the Ss offered the maximum number of 16 sets (there were 16 spaces provided on the answer sheet) on the immediate, as opposed to the five-week retention R test ($\chi^2=13.90$, $df=1$, $p<.001$). There were 13, 14, 13, and 16 in the NP-NC, NP-C, P-NC, and P-C groups, respectively, who did so on the immediate R test, but only 5, 9, 8, and 6, respectively, on the retention R test. Although the chance generation of correct solution sets was possible, the probabilities involved were too low to have greatly affected the degree of forgetting noted.

The time allowed did not appear to affect the correct scores to any great extent. If the Ss were able to solve the problems, they were able to do so in the ten minutes allowed. Although the time scores did not appear to be systematically affected by prerequisite and/or criterion practice, there was a tendency for the P-C Ss to take less time than the NP-NC Ss on the immediate tests and more time on the five-week tests. The reason for these results was not clear, but they probably reflect a confounding of unspecified motivation and learning factors. All of the time means were higher than they would otherwise be since many of the unsuccessful Ss simply waited out the ten-minute test period without any concerted effort.

DISCUSSION

Under the conditions of this study, prerequisite practice facilitated R problem solving. The effect of this variable was somewhat smaller on the RG and smaller still on the N transfer problems. Since the information presented at the criterion level directly related to the R problems, less to the RG problems, and even less to the N problems; these results suggest that the effects of prerequisite practice were largely via criterion learning. Practice in translating card properties into symbols and writing symbol sets associated with single labeled cards could well have made it easier to learn the criterion material since continual reference was made to relationships between cards and symbol sets at the latter level. Increasing prerequisite learning, through practice, could have effectively reduced the number of new concepts with which the learner had to deal at the criterion level. Having well learned the single card-symbol set relationships, primary attention could have been given the new notions such as "finding the collection of all possible sets associated with each of several cards."

In effect, it was felt that these results primarily reflect the relationships between the prerequisite and criterion material. If these materials had been independent of one another, the effects due to prerequisite practice might not have obtained (e.g., see Reynolds and Glaser, 1963). A limitation of this study was that these relationships were not systematically varied, they were merely identified as potentially critical boundary conditions. More definitive studies await greater sophistication in the analysis of subject matters in terms of their behavioral invariants (e.g., see Gagné, & Brown, 1961; Gagné, et. al., 1962; Scandura, 1965; Stolurow, 1964).

Since only one order of testing was used, the lack of any clear cut transfer effects can be given an alternate explanation. The practice provided while solving the test R (and RG) problems may have lessened the effects of the treatment variables on RG (and N) performance, particularly that of criterion practice. Although a weak sequence effect of this sort has been observed (Scandura, 1965), it did not seem likely that controlling for this factor would have appreciably affected the critical results due to prerequisite practice. Undoubtedly, the R problems would have reflected the group differences most and the N problems least.

The lack of treatment effects after five weeks did not necessarily reflect differential forgetting rates. Both the previously mentioned asymptotic conditions on the retention test and testing each S twice would have tended to reduce the treatment effects. The Ss were allowed 30 (plus 4 on the N criteria) minutes of problem-solving practice (without knowledge of results) during the original testing period. Practice during the learning phases of the experiment differed by, at most, 22 minutes.

It is important to note that nothing definitive can be said about the relative effects of prerequisite and criterion practice even though the times

allowed for each were identical. The relative effects probably depend on the difficulty of the material, exactly what is practiced, and the dependent measures used. Nonetheless, the present study demonstrates that there are, at least, some conditions under which prerequisite practice has a greater effect on problem-solving performance than an equal amount of practice time at the criterion level. Since no attempt was made to specifically favor prerequisite practice and since the card material (Scandura, 1964a) was constructed so as to reflect certain kinds of mathematical problems, it would appear safe to conclude that teachers would be ill-advised to gloss over preliminary topics so as to spend more time on the main ideas, particularly when the preliminary topics are relatively unfamiliar to the students. This problem is a very real one in many college classrooms, particularly in the technical and scientific areas.

References

Ausubel, D. P. The psychology of meaningful verbal learning. New York: Grune & Stratton, 1963.

Ausubel, D. P. and Fitzgerald, D. Organizer, general background and antecedent learning variables in sequential verbal learning. J. educ. Psychol., 1962, 53, 243-249.

Ausubel, D. P. and Fitzgerald, D. The use of advance organizers in the learning and retention of meaningful verbal learning. J. educ. Psychol., 1960, 51, 267-272.

Brownell, W. A. The effect of unfamiliar settings on problem solving. Duke Univer. Res. Studies in Educ., 1. Duke Univer. Press, 1931, 86.

Bruner, J. S., Goodnow, Jacqueline J., and Austin, G. A. A study of thinking. New York: Wiley, 1956.

Gagné, R. M. The acquisition of knowledge. Psychol. Rev., 1962, 69, 355-365.

Gagné, R. M. and Brown, L. T. Some factors in the programming of conceptual learning. J. exp. Psychol., 1961, 62, 313-321.

Gagné, R. M., Mayor, J. R., Garstens, M. L., and Paradise, N. E. Factors in acquiring knowledge of a mathematical task. Psychol. Monogr., 1962, 76 (Whole No. 526).

Gagné, R. M. and Paradise, N. E. Abilities and learning sets in knowledge acquisition. Psychol. Monogr., 1961, 75 (Whole No. 518).

Kersh, B. Y. Variations in number symbols and instructional procedures in learning numerical concepts. Unpublished dissertation, Berkeley, 1955.

Reynolds, J. H. and Glaser, R. Repetition and spaced review in the learning of connected discourse. Washington, D. C.: U. S. Office of Education, 1963. (Tech. Rep. 1343).

Scandura, J. M. Abstract card tasks for use in problem solving research. J. exp. Educ., 1964, 33, 145-148. (a)

Scandura, J. M. An analysis of exposition and discovery modes of problem solving instruction. J. exp. Educ., 1964, 33, 149-160. (b)

Scandura, J. M. Algorithm learning and problem solving. Washington, D. C.: U. S. Office of Education, 1965. (Tech. Rep. S-097).

Stolurow, L. M. A taxonomy of learning task characteristics. Washington, D. C.: Office Tech. Services, Dept. Commerce, 1964. (Tech. Rep. AMRL-TDR-64-2).

White, H. M. Does experience in the situation involved affect the solving of a problem? Education, 1934, 54, 451-455.

TABLE 1

Summary of Means and (Standard Deviations)

Immediate						Five Weeks					
R		RG		N		R		RG		N	
Correct	Time	Correct	Time	Correct	Time	Correct	Time	Correct	Time	Correct	Time
				NP-NC							
2.7	8.3	2.8	7.6	2.6	7.7	1.4	6.9	1.2	6.3	.8	6.2
(3.6)	(2.9)	(2.9)	(2.8)	(2.7)	(1.8)	(1.6)	(3.1)	(1.8)	(3.4)	(1.6)	(3.3)
n=20		n=20		n=20		n=19		n=19		n=19	
				NP-C							
3.2	8.9	2.9	8.0	1.8	7.5	2.2	8.2	1.6	7.0	1.3	7.8
(4.8)	(2.1)	(2.4)	(2.6)	(1.8)	(2.7)	(4.1)	(2.6)	(2.5)	(3.5)	(2.0)	(3.2)
n=20		n=20		n=20		n=20		n=20		n=20	
				P-NC							
4.0	7.6	4.2	7.9	1.9	8.3	2.1	6.8	2.2	5.5	.9	5.1
(4.1)	(2.7)	(3.2)	(2.4)	(2.7)	(2.3)	(2.6)	(2.8)	(2.2)	(3.1)	(1.3)	(3.4)
n=20		n=20		n=20		n=20		n=20		n=20	
				P-C							
6.5	6.4	4.1	7.0	3.2	7.3	2.7	8.2	2.4	6.9	1.4	7.8
(5.9)	(2.3)	(3.1)	(2.5)	(3.2)	(2.1)	(4.6)	(1.8)	(3.1)	(2.9)	(1.7)	(2.9)
n=20		n=20		n=20		n=19		n=19		n=19	

TABLE 2

Anovas on Correct Scores

Source	df	R MS	R F	RG MS	RG F	N MS	N F
Bet. prereq. (P)	1	82.52	3.054...	45.32	3.755..	2.13	< 1
Bet. crit. (C)	1	50.95	1.886	.32	< 1	5.15	< 1
P x C	1	6.74	< 1	.32	< 1	11.61	1.426
Pooled bet. Ss (error)	72	27.02		12.07		8.14	
Bet. ret. (R)	1	172.66	47.565***	106.11	41.288***	65.79	29.905***
P x R	1	32.24	8.882**	1.90	< 1	2.13	< 1
C x R	1	5.91	1.628	1.50	< 1	.95	< 1
P x C x R	1	13.93	3.837	.93	< 1	9.50	4.318*
Pooled R x Ss (error)	72	3.63		2.57		2.20	

*sig. at .05 level

**sig. at .01 level

***sig. at .001 level

Algorithm Learning and Problem Solving

Joseph M. Scandura

Florida State University

Abstract

Four groups of 21 Ss were presented with an algorithm (A) and tested on routine (R), generalization (RG), and novel (N) problems. The A Ss had no prior training; the SA Ss were presented with information (level S) deemed necessary for learning the R algorithm; the SCA Ss also were presented with information (level C) deemed useful in modifying the R algorithm (so as to solve the RG and N problems); the SCPA Ss, in addition, had the problems defined and were presented with relationships between an illustrative R problem and its solution (level P). The SA Ss performed better than the A Ss on the R and RG problems ($p < .001$); there were no other significant increases due to amount of information presented. Only the SA-SCA RG difference resulted in a t greater than 1. On the N problems, only the SA-SCA difference was significant ($p < .05$), the A-SA difference resulted in a t of 1.44. Successful problem solving did not depend on an understanding of the problem involved. Transfer was attributed to learning syntactic constraints (principles) relating the algorithm and the problem characteristics. The results also demonstrated the feasibility of predicting problem solving performance by subjectively analyzing structural relationships between the criterion and the information presented. Implications were discussed.

ALGORITHM LEARNING AND PROBLEM SOLVING[1]

Joseph M. Scandura
Florida State University

Algorithms exist for solving many types of problems. The step-by-step computational procedures used in arithmetic provide perhaps the most familiar examples, but algorithms are also used in dealing with all sorts of practical and theoretical problems--from "trouble shooting" to mathematical physics. A common feature of such procedures is that they can be applied mechanically without "understanding."

Many educators and subject matter specialists would maintain that algorithms are limited in their usefulness to specific problem situations and that transfer to variants of the original problems requires an understanding of underlying principles. The fact that understanding is typically defined in terms of performance on some transfer task (Smedslund, 1953), however, poses a problem for researching such conjectures. To operationally define understanding in terms of problem solving transfer, would clearly lead to circularity. One way to overcome this difficulty is to operationally define "understanding" in terms of the amount of information presented.

This study was designed to determine whether given information about problem structure and/or subtasks is necessarily reflected in problem solving performance. It was hypothesized that such information facilitates algorithm learning only when the information is specifically needed to perform the algorithm. Other information about problems reflects itself only to the extent that the algorithm is inadequate to solve transfer problems and to the extent that the information provides a basis for additional positive transfer to such problems. In particular, definition of the problems, themselves, may

have little effect on transfer whereas generalization of the algorithm taught may provide a profitable mode of attack.

METHOD

Materials and subjects.- The materials were based on some abstract materials (Scandura, 1964a) developed for use in problem solving research. The material was based on an extension of card tasks frequently used to study concept formation (e.g., Bruner, Goodnow, and Austin, 1956). Sixty-four cards were used. Each card had two objects on it, six two-valued attributes in all. Corresponding to each of the 12 attribute values was a common symbol. Each card was assigned one of several possible labels. A symbol set was said to be associated with a given labeled card if, and only if, the symbols in the set were related to the card properties in a specified way. A card problem consisted of determining all symbol sets associated with each card comprising the problem. There was an algorithm for solving each of the many possible problem types. Problem types differed in the card labels used, and/or the algorithm pattern and number of solution sets.

For the purposes of this experiment, four levels of the material were identified. At level one (Symbol) were the twelve one-to-one correspondences between the card properties and symbols which represented those properties. There was a common principle relating each property to the corresponding symbol. For example, S_1 and T_2 stood for the first object is Small and the second is a Triangle, respectively. Level two (Criterion) consisted of labeling the cards (either +all or +1) and defining corresponding (criterion) principles by which it was possible to decide, for each set of symbols, whether it was or was not associated with any given labeled card. A set of

symbols was associated with a +all (+1) card if, and only if, all (exactly one) of the symbols corresponded to the card properties. Level three (Problem) consisted of defining the problems (i.e., finding all symbol sets associated with each of the three labeled cards comprising a problem) and pointing out some of the relationships between an illustrative problem (see R problem below) and its solution. For example, every symbol used in writing the solution sets corresponded to some property of each of the +all cards and the symbol which corresponded to the +1 card property appeared in each solution set. In no case was information given as to how the solution was obtained. Level four (Algorithm) consisted of an efficient algorithm for solving the R problems. There were four operations involved in each of the algorithms used: (1) writing those six symbols corresponding to the properties of the first (+all) card, (2) crossing out some of these symbols according to the second card, (3) encircling and coding one or more of the remaining symbols according to the third card, and (4) writing all solution sets on the basis of the algorithm pattern resulting from the first three operations.

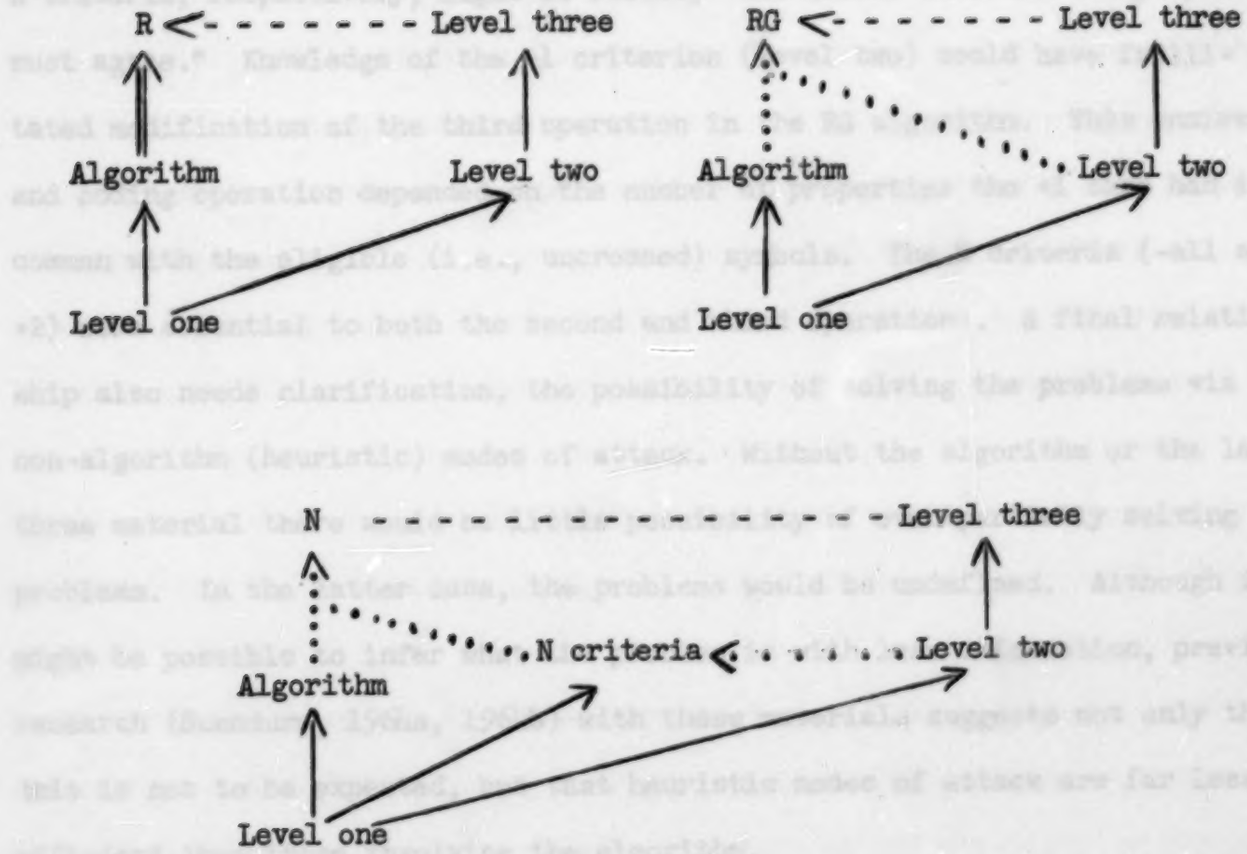
In addition to the R problems, two other types were considered. The routine generalization (RG) problems also consisted of two +all cards and one +1 card, but successful RG problem solving required a relatively minor modification of the algorithm taught (i.e., the algorithm pattern was either $- \overset{1}{\theta}\ \theta\ \theta$ or $- \overset{1}{\theta}\ \theta -$ as opposed to the $- - - \overset{1}{\theta}$ pattern used for the R problems.[2] The two novel (N) problems each consisted of a +all, -all, and +2 card. The algorithm patterns were $\overset{2}{\theta}\ \theta\ \theta\ \theta$ and $- \overset{2}{\theta}\ \theta\ \theta$. Successful RG performance depended primarily on the valid generalization of the third and fourth algorithm operations whereas N problem solving involved modification of the second operation as well. A corresponding set of retention problems

was constructed. Each of the five algorithm patterns (one R, two RG, and two N) was represented. A statement of the -all and +2 N criteria, each with one associated symbol set as an illustration, also was prepared.

The level one material was used in the description of all of the other levels, including the algorithm; the level two material was used only in describing the level three material; but, neither the level three nor level four material was prerequisite to any of the other levels. The R- algorithm could be performed without knowledge of either level two or level three. The levels were related to each other and to the test problems as indicated in Figure 1.

Insert Figure 1 about here

Since the structural relationships indicated in Figure 1 played a central role in the present research, the bases for these analyses deserve further explication. Two of the relationships indicated, use of the material in presenting new material and sufficiency of the algorithm, have a largely objective basis. The former relationship was indicated only in those cases where the information presented at one level was actually used in the description of the higher order material. An example was provided by the use of symbols to describe and illustrate the +1 criterion. Sufficiency was indicated in that the algorithm provided a sequence of mechanical steps by which any R problem could be solved. Other than reading skills and other general abilities only knowledge of the symbol-property relationships was necessary to follow the procedure. Transfer potential was more subjectively based. Judgments were made as to whether the levels involved were related by a common principle (s). Learning the subtask above the arrowhead (see Fig. 1) could be facilitated by



- - - indicates possibility of solving problems via non-algorithm (heuristic) mode of attack

. . . indicates potential positive transfer by means of structual similarities or relevancy of information for successful problem solving

——— indicates use of material in presenting new material

═══ indicates sufficiency of algorithm for (R) problem solving

Fig. 1. Relationships between the R, RG, and N problems and the prerequisite material.

prior acquisition of the common principle (s) during the learning of the other subtask. The principle relating the +1 and +2 criteria at level two and N criteria, respectively, might be stated, "the number tells how many symbols must agree." Knowledge of the +1 criterion (level two) could have facilitated modification of the third operation in the RG algorithm. This encircling and coding operation depended on the number of properties the +1 card had in common with the eligible (i.e., uncrossed) symbols. The N criteria (-all and +2) were essential to both the second and third operations. A final relationship also needs clarification, the possibility of solving the problems via non-algorithm (heuristic) modes of attack. Without the algorithm or the level three material there would be little possibility of even partially solving the problems. In the latter case, the problems would be undefined. Although it might be possible to infer what the problem is with less information, previous research (Scandura, 1964a, 1964b) with these materials suggests not only that this is not to be expected, but that heuristic modes of attack are far less efficient than those involving the algorithm.

The experimental materials consisted of a one to two page double spaced introduction to each level of the material, a number of practice problems (enough to keep even the fast Ss busy throughout the corresponding practice period) with answers on the following page, and some routine arithmetical and algebraic factoring problems which served as filler. The Ss were 84 eleventh grade college preparatory mathematics students at the Kenmore East High School in Kenmore, New York. None of these Ss was in an honors section.

Design and Procedure.- The amount of prerequisite material was varied. After having been shown and having practiced different amounts of relevant prerequisite material, each of four groups were presented with the algorithm

and given an opportunity to practice using it. The total time spent on the algorithm was 17 minutes. To control for possible massing and fatigue effects, all Ss had approximately 39 minutes of preliminary work (including the prerequisite instruction and/or filler material). Group A was given 39 minutes of filler before the Algorithm was presented. Group SA had 8 minutes to learn and practice the level one (Symbol) material and 31 minutes of filler before the algorithm was presented. Group SCA had 8 and 14 minutes to learn and practice the level one and level two (Criterion) material, respectively, and 17 minutes of filler before the algorithm was presented. Group SCPA had 8, 14, and 17 minutes to learn and practice the level one, two, and three (Problem) material, respectively, before the algorithm was presented. A table of random numbers was used to assign the 84 Ss equally to the four treatment groups.

All of the materials were presented by means of dittoed booklets and the instructions were given by tape recorder for uniformity. The presentations at each level consisted of both a concise statement of and questions or problems based directly on the material. Filler material was used to replace entire levels, not merely the practice period within a level. All Ss were run at the same time. Three different classrooms were used, but each treatment group was equally represented in each room.

Immediately after the learning period and four weeks later, all Ss were given ten-minute R, RG, and N tests, each consisting of two problems.[3] Three testing orders were used in each treatment group--(R, RG, N), (RG, N, R), and (N, R, RG). The -all and +2 criteria were presented just prior to the N testing (3 minutes). Total learning and testing time (89 minutes) was the same for all treatment groups.

The elapsed time was put on a blackboard at 15 second intervals and the Ss were required to record their times on each test. Whenever an S failed to record his time, he was given a score of 10 (minutes).

RESULTS AND DISCUSSION

Differences due to class and testing order were minimal, so that these factors were ignored in the subsequent analyses. The only tendency noted was that RG (and N) performance was slightly enhanced (not significantly) when the R (and RG) problems came first, possibly due to learning during the R (and RG) test period (s).

Due to the non-normality evident within the treatment groups on most measures and heterogeneity of some of the variances, the conservative Welch approximation test (Winer, 1962, 37-38) was used to test the hypothesized relationships.

R Problems.--The number of correct solution sets and the total number of sets offered (including incorrect sets) ranged from zero to 16 (100%). Time spend on the R problems ranged from 1-3/4 minutes to the full 10 minutes allowed. A summary of all means and standard deviations is given in Table 1.

Insert Table 1 about here

The difference between the A and SA groups (number correct) on the immediate R test was highly reliable ($t=5.248$, $df=20$, $p < .001$) whereas none of the SA, SCA, and SCPA paired-comparisons approached significance ($t < 1$). There were 19, 1, 0, and 3 zero scores and 1, 7, 9, and 5 perfect (16) scores in the four groups, respectively. These results conformed to expectations.

According to Figure 1, the level one material was necessary for successful application of the algorithm; levels two and three were not.

Twelve A Ss offered no solution sets, but all except one of the remaining Ss offered some. There were 6, 11, 16, and 16 Ss respectively, who offered 16 sets (the total number of spaces present on the answer sheet). These results are not surprising since the symbol meanings had not been presented to the A Ss. Even the SA Ss had had little practice in writing symbol sets, and none with labeled cards, prior to level A. Fifteen of the A Ss spent the full ten minutes on the R test whereas 4, 2, and 2 of the other Ss did, respectively.

The retention data showed essentially the same pattern. Only the A-SA mean difference (number correct) was significant ($t=2.937$), $df=15$, $p < .02$). The relatively large amount of forgetting in the SA, SCA, and SCPA groups may have been more apparent than real. These Ss had something to forget. The performance of the A Ss, on the other hand, was minimal (19 of 21 had zero scores) to begin with. The slight A-group improvement noted on the retention test can probably be attributed to the repeated testing.

As can be seen from Table 1, there were also some significant time differences. Those effects, however, probably reflect more than degree of learning even though the A Ss consistently took more time on the tests than the others. In particular, amount of information about the problems may have interacted with the success achieved on the algorithm level practice problems to produce differential test motivation levels. The algorithm was completely new to the A Ss so that extinction may not have progressed as far, for example, as in the SCPA group (for the relatively unsuccessful Ss). The time scores could have reflected such motivational effects, together with degree of learning, making interpretation difficult.

RG Problems.- Overall, the Ss performed better on the R problems than on the RG problems. Whereas no S achieved a perfect RG score (14), almost as many Ss wrote a symbol set in each of the 16 spaces provided as did on the R problems. As suggested in Figure 1, the A-SA difference (number correct) was significant ($t=4.033$, $df=20$, $p < .001$), but none of the paired-comparisons between the SA, SCA, and SCPA means were. The SA-SCA difference, however, was suspect ($t=1.209$). According to the method-section-discussion, such an effect would have been attributed primarily to familiarization with the +1 criterion.

Whereas the SCA-SCPA R retention mean difference was positive, the corresponding RG difference was in the opposite direction. Neither difference, however, was significant.

N Problems.- A somewhat different pattern of results obtained with the N problems. Whereas the A-SA difference (number correct) was not significant ($t=1.436$), the SA-SCA difference was ($t=2.148$, $df=20$, $p < .05$). The SCA mean was slightly greater than the SCPA mean, but again the difference was not significant. Although all of the hypothesized effects were not significant, Figure 1 adequately represents the magnitude and direction of the N mean differences. The SCA (and SCPA) Ss had three potential positive transfer routes whereas the SA Ss had only two. Apparently, learning the +all (+1) criteria significantly facilitated -all (+2) learning.

After four weeks, only the A-SCPA difference was significant ($t=2.370$, $df=18$, $p < .05$). The SCPA-SCA retention mean difference, although not significant, favored the SCPA group proportionately to a greater extent than did the corresponding R and RG differences. It was not possible to test this effect due to experimenter error. That part of the instructions tape asking the Ss to sign their names during the retention session was accidentally erased and

was not detected until after the experiment was completed. It is interesting, nonetheless, to speculate as to why the relatively long level three period (17 minutes) might have increased N, but not R, retention after four weeks. These effects could have been due to the increased practice afforded, an increase in the meaningfulness of the problems, competition between algorithmic and heuristic modes of attack, or all, in some unknown combination. Since N problem solving required more than mastery of the algorithm taught, it is likely that meaningfulness was involved to some extent. Although the relationships were not at all clear, these retention effects may have been related to the recent finding of Gagné and Bassler (1963) that subordinate skills were forgotten in a seemingly unsystematic fashion, even when the Ss were able to perform satisfactorily on the superordinate tasks.

The number of sets offered and the time taken to complete the N tests paralleled the results on the tests. The differences, however, were generally smaller.

General comments.- These results demonstrate that successful problem solving does not necessarily depend on an understanding of the problem involved. Only the SCPA Ss were presented with a definition of the problems, but this information was not reflected in problem-solving performance. Perhaps even more surprising was the finding that transfer also did not depend on "understanding." Of course, it was possible that the successful non-SCPA Ss may have inferred what the problems were during the testing and/or the level four presentation. If so, none of the Ss were able to state any reasonable facsimile of the level three definition. Judging from the comments made, it was more likely that the successful Ss had detected relationships among the algorithms and were able to modify them according to the syntactical constraints

present between the algorithm and the individual problem characteristics. For example, the -all label may have induced the Ss to cross out those symbols which corresponded to the card properties rather than to eliminate symbols as when the "opposite" +all label appeared. Similar constraints could have made it possible to deal effectively with the RG problems.

Understanding in the subject matter areas has never been adequately specified. Whereas it may be sufficient to operationally define understanding in terms of a generalized sort of transfer in many situations, the use of algorithms raises new problems. Should understanding reflect the degree to which problem elements and the goal are meaningful in terms of Ss prior knowledge (as, for example, in the level three definition) or should it reflect the attainment of pattern constraints among the problem-solving procedures? These are different, yet equally valid, conceptions of understanding. The major implication, of course, is that the assessment of one or the other of these "kinds of understanding" depends critically on the transfer measure used. As suggested by Cronbach (1956), global assessment involves making hazardous inferences and can only confound such issues, never clarify them. Subject matter, particularly language and mathematics, generally involves both semantic and syntactic constraints. A limitation of this study was that no attempt was made to explicitly separate these factors. Further clarification of the distinctions between semantic and syntactic constraints and the development of techniques for determining the acquisition of such abilities would be an important milestone in educational science.

Another facet of this research deserves mention. These results demonstrate the feasibility of predicting problem-solving performance on the basis of a subjective analysis of the structural relationships between the criterion

task and the information presented. The general approach did not appear to be limited to problem solving. There was no intrinsic reason why similar analytical procedures could not be applied to the learning of new material as well. In addition, there was no reason why the Ss' background knowledge could not equally well have been assessed. Intensive, rather than extensive, testing procedures could be used to determine what was known that might relate to each class of problems. Combined with structural analyses, such information could then be used to provide information as to how S might perform on the criterion tasks. More precise prediction awaits a more complete understanding of just what is involved in such assessment and analyses.

The approach used in the present study to determine structural relationships was reminiscent of task analysis procedures which have been popularized, in education, by Gagné (e.g., 1962). Instead of asking, however, "What would S need to know in order to perform this task?" as Gagné did, the present form of analysis was concerned with the question, "Could this information facilitate problem solving and, if so, how?" At present, both sorts of analyses can be evaluated only in terms of how well they predict the data. It would be highly desirable to have an independent measure.

The form of structural analysis used was at once more general and less precise than task analysis. It was more general in that it provided for prediction even though the information presented did not appear to be sufficient for successful criterion performance and less precise in that no attempt was made to identify molecular subtasks. Although task analysis is currently being employed in devising new science curricula (e.g., see Science in the kindergarten and early grades), a more molar approach may be more feasible. It is obviously impossible to include in a curriculum everything S needs to

know about a particular topic. Relatively simple structural analysis procedures may make it possible for the educational psychologist, in conjunction with subject matter specialists, to explore the problem-solving implications (assuming problem-solving ability is a desired objective) of a particular set of curricular materials before the costly tasks of formal evaluation and implementation are undertaken. Of course, this approach may lend itself more naturally to some areas, like mathematics, than to others.

References

Bruner, J. S. Goodnow, Jacqueline, and Austin, G. A study of thinking. New York: Wiley, 1956.

Cronbach, L. Assessment of individual differences. Annu. Rev. Psychol., 1956, 7, 173-196.

Gagné, R. M., and Bassler, O. C. Study of retention of some topics of elementary nonmetric geometry. J. educ. Psychol., 1963, 54, 123-131.

Gagné, R. M. The acquisition of knowledge. Psychol. Rev., 1962, 69, 355-365.

Scandura, J. M. Abstract card tasks for use in problem solving research. J. exp. Educ., 1964, 33, 145-148.

Scandura, J. M. An analysis of exposition and discovery modes of problem solving instruction. J. exp. Educ., 1964, 33, 149-160.

Scandura, J. M. Problem solving and prior learning. Washington: U. S. Office of Education, proj. no. S-097.

Science in the kindergarten and early grades. Science Education News. 1963, 12 p.

Smedslund, J. The problem of "what is learned?" Psychol. Rev., 1953, 60, 157-158.

Winer, B. J. Statistical principles in experimental design. New York: McGraw, 1962.

Footnotes

[1]This research was supported by the Cooperative Research Program of the U. S. Office of Education, Project S-097. Thanks are due Robert Magee for his help with both the preparation of the materials and the conduct of the experiment and to Richard Eiswirth for help with the data analysis.

[2]Dashes (-) represent symbols used in solution sets and encircled dashes (⊖) represent those symbols that correspond to ±1 or ±2 card properties. For more details see the article by Scandura (1964a).

[3]Reliability estimates of .90, .61, and .65 for the R, RG, and N tests, respectively, were obtained by computing the correlation coefficients between the correct scores on the two problems in each test. The problems were randomly designated "X" or "Y" so that half of the X(Y) scores were from problem one and half from problem two.

TABLE 1

Means and (Standard Deviations) on Number Correct Scores--Immediate Tests

			R			RG			N	
		Correct	Total	Time	Correct	Total	Time	Correct	Total	Time
(n=21)	A	$\bar{x}$	.81	5.19	8.37	.24	5.67	8.13	.14	4.81	8.63
		$\sim$	3.52	7.28	2.70	.54	7.57	2.65	.48	6.89	1.91
(n=21)	SA	$\bar{x}$	9.05	13.19	5.20	2.62	11.67	5.11	.67	6.43	7.76
		$\sim$	6.29	4.32	2.77	2.65	5.33	2.25	1.62	5.98	1.80
(n=21)	SCA	$\bar{x}$	10.24	14.62	4.79	3.67	14.05	6.06	2.29	9.43	7.24
		$\sim$	6.43	3.07	2.41	2.97	3.43	2.67	3.05	6.00	2.52
(n=21)	SCPA	$\bar{x}$	9.29	15.33	5.60	3.95	13.57	5.14	2.19	12.86	7.92
		$\sim$	5.87	1.50	2.17	3.22	3.31	2.14	3.06	4.30	2.08

TABLE 2

Means and (Standard Deviations) on Number Correct Scores--Four Week Tests

		R			RG			N		
		Correct	Total	Time	Correct	Total	Time	Correct	Total	Time
(n=19) A	$\bar{x}$	1.74	5.05	8.50	.74	5.68	8.13	.37	4.84	8.00
	$\sim$	5.03	7.07	2.70	1.92	7.18	3.00	1.61	7.26	3.34
(n=16) SA	$\bar{x}$	6.88	10.50	6.81	2.00	11.00	5.10	.69	6.88	6.12
	$\sim$	6.26	6.78	2.75	2.81	6.29	3.09	1.96	6.77	3.63
(n=18) SCA	$\bar{x}$	6.39	13.67	5.33	2.44	12.83	6.78	1.28	11.22	7.81
	$\sim$	7.39	4.04	1.72	3.60	4.79	3.22	2.78	5.93	3.16
(n=21) SCPA	$\bar{x}$	4.81	13.67	6.96	2.95	13.19	4.50	2.33	12.00	4.78
	$\sim$	5.19	5.21	2.17	3.61	5.74	2.40	3.39	6.10	2.41

J. exp. Educ. in press

ABSTRACT

Prior Learning, Presentation Order, and Prerequisite Practice in Problem Solving

Joseph M. Scandura

Florida State University

Abstract hierarchically ordered materials were presented in modified program form to 165 college Ss. Symbols used to denote prerequisite notions were used in defining the problems. Three variables; amount of prelearned prerequisite material, presentation order (prerequisite-criterion or criterion-prerequisite), and prerequisite practice; were manipulated in a 2 x 2 x 2 factorial design. Three additional groups served as controls. Ss were tested on the prerequisite material as well as on training and transfer problems. The major results were that problem solving performance was improved: (1) by the pre-experimental availability of prerequisite material, even after repeated re-introduction of criterion level materials, hints, and practice in problem solving, (2) when the prerequisite material was presented prior to the criterion material -- this effect disappeared after the re-introduction of the criterion material, and (3) by prerequisite practice only when the prerequisite material came first -- this effect was relatively weak and transitory. The relationship between the prerequisite and criterion materials was deemed crucial.

Prior Learning, Presentation Order, and Prerequisite Practice in Problem Solving[1]

Joseph M. Scandura

Florida State University

This is the third in a series of experiments dealing with problem solving and prior learning. In each of these experiments, the Ss were presented with two (or more) hierarchically related aspects, prerequisite (P) and criterion (C), of some abstract card material (Scandura, 1964a; 1964b) and were tested on (training) problems defined at the C level. Both the P and C materials were presented via ordinary discourse. The words, symbols, and/or graphic materials used to denote P notions were also used in presenting the C material. The C material consisted of a definition of the training problems, an illustrative problem, its solution, and pointing out relationships between the problem and its solution. Performance was tested on both the training problems, and new, though related, transfer problems.

In experiment one, Scandura (1965a) found that practice, with feedback, at the P level facilitated problem solving performance whereas practice in solving the problems themselves did not. These effects were attributed to increasing familiarity with P denotants thereby facilitating C learning. In experiment two (Scandura, 1965b), exposure to information about the problems, coupled with related practice, reliably improved problem solving performance, via an algorithm, but only when the information was either specifically used to describe the algorithm or clearly provided a basis for modifying the algorithm taught so as to successfully attack transfer problems. Other information about the problems, and particularly definition of the problems, in terms of component elements, did not facilitate problem solving in any case. Although no direct comparison was possible, the effect of exposure with practice in experiment

two appeared to be considerably stronger than the experiment one effect due to practice alone.

The present study was concerned with three variables: (1) amount of relevant prior P knowledge, (2) order of presenting the P and C material, and (3) prerequisite practice.

Previous studies (e. g., Gagné and Paradise, 1961; Gagné, Mayor, Garstens, and Paradise, 1962) have shown that there is a strong relationship between P achievement and C learning. Successful performance on C tasks almost never obtained without the prior acquisition of all prerequisites. Nonetheless, in these studies degree of P learning was not experimentally manipulated; as such, it was not demonstrated to be a causal factor in improving C test performance. Some unknown factor may have concomitantly improved performance on both the P and C tests. In the present study, P learning was an independent, rather than dependent, variable. This study was not, however, simply concerned with comparing the effects of prelearning versus no prelearning. The question asked was whether mere exposure to abstract P material is sufficient for successful C test performance. This question is an apparent assumption in many college mathematics classrooms.

The second variable of concern was the order of presenting the P and C material. In the previously cited studies (Scandura, 1965a, 1965b), a proaction paradigm was used. The experimental Ss were presented first with Task A (with practice), then Task B, and finally were tested on Task B. The control Ss were either given Task A without practice or were not shown Task A. Although it was postulated that the relationship between Tasks A and B may have been a critical boundary condition for the experimental findings, the design did not allow definitive statements in this regard. The P material may have improved

problem solving directly and not by mediating C learning. Comparing the two possible orders of presenting the P and C material would settle the issue. If P learning facilitates C learning then the presentation order PC should result in better problem solving performance than the order CP.

The effects on problem solving of P practice were also determined. In accordance with the preceding discussion, P practice was hypothesized to affect problem solving by increasing P learning only in the presentation order PC. In effect, this aspect of the study represented an attempt to both replicate and extend Scandura's (1965a) earlier finding. Replication seemed particularly desirable since other investigators (Reynolds and Glaser, 1963; Gagné et al, 1962) have not found a reliable practice effect. There were, however, some potentially important differences between these studies. In the Reynolds and Glaser (1963) experiment, the materials were not hierarchically ordered. This may be one critical boundary condition. The materials used in the Gagné et al (1962) experiment were programed in small steps; the presentation did not involve extended discourse as in the Scandura (1965a) study. The use of P terms to describe C material may be another important boundary condition.

Problem solving performance was determined immediately after the instruction and repeatedly thereafter with feedback, reintroduction of the C material, and hints intervening. This procedure made it possible to determine the effects of the independent variables under different conditions of familiarity with the C material and the problems. Transfer was also assessed.

METHOD

Materials and Subjects.-- The materials were developed specifically for use in problem solving research (Scandura, 1964a). They represent an elaboration of card tasks frequently used to study concept formation (e.g., Bruner, Goodnow, and Austin, 1956). The basic elements involved are pairs of objects to which labels are attached. Each pair of objects had six properties; there were six two-valued attributes in all. A common symbol was used to designate each of the twelve attribute values. Object pairs were assigned one of three labels, +all, -all, and +1. A set of one or more symbols was said to be associated with a given labeled-object-pair if and only if the symbols in the set were related to the object-pair properties in a specified way. The problems consisted of determining all symbol sets associated with each of three object-pairs comprising a problem. There is an algorithm for solving each of the possible problem types. Problems differed in the labels used, the algorithm pattern (Scandura, 1964a) and/or the number of solution sets.

For the purposes of this experiment, three levels of the material were identified. The object-pairs and symbols introduced at level one were used in describing the level two and three material. The level two denotants were used to describe level three. At level one were the twelve one-to-one correspondences between the object-pair properties and the symbols which represented those properties. There was a common principle relating the properties and symbols. For example, S_1 and T_2 stood for the first object is Small and the second is a Triangle, respectively. Level one practice consisted of translating five written statements, describing object-pairs, into symbols. Level two consisted of labeling the cards, either +all or +1, and defining criterion principles by which it was possible to decide,

for each set of symbols, whether it was or was not associated. A symbol set was associated with a +all (+1) object-pair if and only if all (exactly one) of the symbols corresponded to the object-pair properties. Level two practice was given in two parts, +all and +1. In each part, Ss were required to give a subset associated with one object-pair and to cross out those of a list of 15 sets which did not go with a second pair. Level three consisted of defining the routine (R) problems (i.e. finding all "solution sets" associated with two +all pairs and one +1 pair) and pointing out some of the relationships between an illustrative R problem and its solution. Each R problem had eight solution sets. Each symbol used in writing solution sets corresponded to some property of both +all pairs; the one symbol, which corresponded to a +1 pair property, appeared in each solution set. The algorithm pattern was - 0 - - (1).[2] In no case was information given, at level three, as to how the solution was obtained. Levels one and two together constituted the prerequisite (P) material. The criterion (C) material consisted of level three.

In addition to seven R problems, two N problems were used to test for transfer. Both N problems consisted of two +all pairs and one -all pair, the first with algorithm pattern - - - and the second with pattern - - - -. Short tests also were constructed to determine the degree of symbol, +all, +1, and -all learning. Knowledge of results (KR) was provided for all P practice and R problems and included both a restatement of the questions or problem and the correct answer(s).

The materials were presented in booklet form. The symbol, +all, +1, criterion material, practice for each unit of information (relevant or filler), KR, R problems, N problems, and all tests appeared in

separate sections. Each of the 36 sections in the booklet were separated by a yellow page indicating the number of the section following. An answer booklet was constructed to correspond to each practice, problem, and test section.

The materials were presented to 165 college Ss enrolled in a mathematics education course for elementary teachers at the Florida State University. Participation in the experiment was a course requirement.

Design.-- The Ss were randomly assigned to one of eleven treatment groups subject to equal cells. Eight of these groups gave rise to a 2 x 2 x 2 factorial design. The three independently manipulated variables were: (1) amount of P material still to be learned during the experimental period (much, M, including the symbol, +all, and +1 material and little, L, including only the +1 material -- the symbol and +all material was well-learned prior to the experiment), (2) presentation order (PC or CP), and (3) P practice (practice, Pr, or no practice, No Pr).

The other three groups were controls. Group MC neither prelearned nor was shown any of the P material, group LC prelearned the symbol and +all material but was not presented with the +1 material, and group CC prelearned all of the P material. These groups were included to determine minimal and maximal effects due to the presence of P material.

Procedures.-- The material was presented, at special evening sessions, involving 44 or fewer Ss. The average session size was about 27 Ss. All but the CC Ss were approximately equally represented at each session. Shortly before completing the experiment, a small, but potentially harmful, defect was discovered in the CC booklets. This group was represented disproportionately at the last session.

All Ss reported for a given session at the same time. The L and CC Ss were pretrained in one room while another proctor discussed unrelated mathematics problems with the remaining Ss in a second classroom.

After pretraining, all Ss were brought together in a single room for the experiment. Directions were given by tape recorder for uniformity. The Ss were informed that the purpose of the experiment was to determine how people learn complex material and they were urged to do their best. They were also told that some material would be given, for study, and that questions or problems, based on that material, would be given for them to answer or solve.

Although the first two experiments, in this series, were experimenter paced, this procedure apparently served no useful purpose. Because it frequently reduces within group variance, self-pacing was used in experiment three. The Ss were directed to record the time, appearing on a blackboard, in their answer booklet as soon as they finished each section. The time was changed at one minute intervals. The Ss were not allowed to return to once completed sections. Although specific directions were given, from one to seven Ss, in each group, failed to record their times correctly. Times were often given at the end of several sections rather than at the end of each section. In these cases, a weighted average, compiled separately for each S, was used to estimate times on individual sections. Since all times were rounded to the nearest minute, it is not felt that this procedure resulted in any serious bias.

The suggested time for completing the booklet was 90 minutes. The Ss were directed to spend at least one hour but no more than two. Five Ss required slightly more than two hours to complete the booklet;

their time was recorded as two hours. The total time spent, including the pretraining, averaged about two hours.

The material presented in the first ten sections for each group is presented in Table 1. Beginning with section 11, the materials were

---

Insert Table 1 about here

---

presented as follows to all Ss: 11, R problem number one (R1); 12, R1 answers; 13, R2; 14, R2 answers; 15, level 3; 16, R3; 17, R3 answers; 18, level 3 with hint A (step one in algorithm, Scandura, 1964a); 19, R4; 20, R4 answers; 21, level 3 with hints A and B (steps one and two in algorithm); 22, R5; 23, R5 answers; 24, level 3 with hints A, B, and C (first three steps in algorithm); 25, R6; 26, R6 answers; 27, level 3 with hints A, B, C, and D (entire algorithm); 28, R7; 29, R7 answers; 30, +1 test; 31, +all test; 32, symbol test; 33, -all criterion; 34, N problem one (N1); 35, N2; 36, -all test.

In all, each S was tested on seven R problems, four criterion and symbol tests, and two N problems. Since the reliability of single R problem measures has been shown to be in the neighborhood of .90 (Scandura, 1965a, 1965b), repeated R testing provided an excellent opportunity to determine the effects of the independent variables under different conditions of R practice with KR, C familiarity, and information about an efficient algorithm (see Scandura, 1965b).

RESULTS AND DISCUSSION

The means (standard deviations) and significant F-values are summarized in Tables 2 and 3.

With one notable exception, the results conformed closely to prediction. The four L groups performed better than the M groups on each of the R and N problems ($p < .001$ or .01). Having a minimum of new prerequisite (P) material to learn during the experimental period resulted in superior problem solving ability. More important, the effect of this variable persisted despite the repeated intervention of additional information assumed to facilitate problem solving. This result was somewhat surprising since incidental learning on the earlier tests almost certainly would have tended to reduce the treatment effects. It was also noted that the availability of well-learned P knowledge (i.e. the +all and symbol material) prior to the experiment resulted in better performance on the +1 and -all criterion tests ($p < .05$ and .01, respectively). Positive transfer, via the -all criterion, may have been instrumental, to some degree, in the improved N performance ($p < .01$).

The effects of presentation order on problems R1 and R2 were reliable ($F = 15.61$ and 7.97, $df = 1/112$, $p < .001$ and .01, respectively). Feedback and the re-introduction of the criterion (C) material, beginning after problem R2, however, steadily reduced this effect on succeeding R problems so that there was no evidence whatsoever of any order effect on problems R6 and R7.

There were only two reliable interaction effects, order by practice on problem R1 and amount by practice on the symbol test. The former result

was important; P practice affected R performance more when the P material preceded the criterion. This effect, however, was short lived; practice had no significant effect on any of the remaining problems. Incidental learning, early in the test period, superseded the effect due to P practice. The other interaction, between amount and P practice probably obtained because the L groups had prelearned the symbol material whereas the M groups had not. Practice was not expected to affect L group performance since performance was already at asymptote.

Practice with knowledge of results improved performance on the symbol and +1 material ($p < .01$), but not on the +all material. It is difficult to say why this was, but it is likely that learnability was a factor. Long experience with the card material (Scandura, 1964a, 1964b, 1965a, 1965b) has shown that Ss have considerably more difficulty with the symbol and +1 material than with the +all material. In the latter case, reading may have been sufficient.

The three control groups provided a basis for determining how much problem solving performance was enhanced by merely reading the P material. The MC, M-PC-No Pr, and CC groups provided the comparison base when much P material was to be learned. Merely reading the prerequisite material did not improve R1 performance as much as prelearning ($p < .01$). Reading, on the other hand, did not reliably improve R1 performance beyond that achieved by the MC group which had not been exposed to the P material at all ($p > .10$). The corresponding little-P-material means were ordered in the same way (CC > L-PC-No Pr > LC) but none of the differences was reliable. Whether reading P material is sufficient for C learning apparently depends on the amount of P material to-be-learned (and its difficulty).

The time analyses probably reflect both persistence and ease-of-learning factors. This being the case, these results are not easy to interpret. In some cases, the time spent on the C material or the problems varied directly with the level of relevant knowledge. The L groups spent more time on the R and N problems ($p < .01$ and .05, respectively), the practice groups spent more time on the R problems only when the presentation order was PC, and the PC groups spent more time on the initial C presentation than did the CP groups, especially when most of the P material had been well-learned prior to the experiment. Why some groups spent more time on the C material, but not on the problems, while other groups did the reverse is not at all clear. To be definitive, future studies need to be devised specifically to answer such questions.

Judging from the comments made, observation of the Ss, and prior experimentation (Scandura, 1965a, 1965b), in which time was controlled, it is doubtful that equalizing time would have greatly affected the results. The unsuccessful Ss probably would simply have wasted the extra time.

GENERAL DISCUSSION

All three independent variables affected routine (R) problem solving. Increasing familiarity with criterion (C) level materials and problems, however, differentially modified the initial effects.

The availability of prior relevant knowledge had a powerful and lasting effect on R problem solving. Other results (Gagné and Paradise, 1961; Gagné, Mayor, Garstens, and Paradise, 1962; Maier, 1930) have also demonstrated the importance of prerequisite attainment on later learning and problem solving. With the possible exception of research with young children, however, none to the author's knowledge has demonstrated the relative permanence of this effect. Although the relationship is at best tenuous, it is entirely conceivable that the sort of phenomena demonstrated here with college Ss is but a more restrictive instance of Piaget's (e.g., see Flavell, 1963) notions about more general requisites for intellectual achievement.

In the present study, repeated efforts to counteract preliminary inadequacies by providing knowledge of results (KR), re-presenting the C material, and giving hints were for naught. It would appear that attempts to bring a poorly prepared group to a predetermined level of proficiency by spending more time on the criterion would be ill-advised. The present results suggest that it might be better to return to underlying notions. In effect, this is the basic, although often implicit, assumption underlying most diagnostic and remedial work.

The reliable order effect indicated, rather conclusively, that the symbol, +all and +1 material was indeed prerequisite to the criterion. Although it may have been possible for a few CP Ss to make sense of the C material, when first presented, it is more likely that what relatively little learning there was resulted from remembering the C material until after the prerequisites were presented. At this point, some of the CP Ss may have been successful in reorganizing the C material in a meaningful way. That order has not been crucial in other studies (e.g., Levin and Baker, 1963; Roe, Case, and Roe, 1962; Roe, Massey, Weltman, and Leeds, 1962) suggests that whatever relationships there were between the units of material to-be-learned were either minimal or balanced out in the different orders used. The effects of order, however, were not permanent in the present experiment. Presumably, the CP Ss were better prepared for C learning the second time they saw the material--between problems R2 and R3.

The positive interaction between P practice and order of presentation also lends support to Scandura's (1965a) earlier hypothesis that the relationship between the P and C material was crucial. This result also suggests that P practice merely increases the degree of P learning, thereby facilitating C learning and problem solving (Scandura, 1965a). The P practice effect obtained, in the present study, even though KR was given only once per unit of material. In the previous study (Scandura, 1965a), more practice was given and the high school Ss were allowed to turn back and forth while working the practice exercises. Practicing the P material after the C material was presented had no appreciable effect on R1 performance--what little effect there was was negative. Even with the order PC, however, the effects of P practice was relatively weak and transitory.

REFERENCES

Bruner, J. S., Goodnow, Jacqueline J., and Austin, G. A. A study of thinking New York: John Wiley & Sons, 1956.

Flavell, J H. The developmental psychology of Jean Piaget. Princeton: Princeton: Van Nostrand, 1963.

Gagné, R. M. and Paradise, N. E. Abilities and learning sets in knowledge acquisition. Psychol. Monogr., 1961, 75 (Whole No. 518).

Gagné, R. M., Mayor, J. R., Garstens, M. L., and Paradise, N. E. Factors in acquiring knowledge of a mathematical task. Psychol. Monogr., 1962, 76 (Whole No. 526).

Levin, G. R. and Baker, B. L. Item scrambling in a self-instructional program. J. educ. Psychol., 1963, 54, 138-143.

Maier, N. R. F. Reasoning in humans: I. On direction. J. comp. Psychol., 1930, 10, 115-143.

Reynolds, J. H. and Glaser, R. Repetition and spaced review in the learning of connected discourse. Washington, D. C.: U. S. Office of Education, 1963 (Tech. Rep. 1343).

Roe, K., Case, H. W., and Roe, A. Scrambled versus ordered sequence in auto-instructional programs. J. educ. Psychol., 1962, 53, 101-104.

Roe, A., Massey, Mildred., Weltman, G., and Leeds, D. Automated teaching methods using linear programs. J. Psychol., 1962, 40, 198-201.

Scandura, J. M. Abstract card tasks for use in problem solving research. J. exp. Educ., 1964, 33, 145-148. (a)

Scandura, J. M. An analysis of exposition and discovering modes of problem solving instruction. J. exp. Educ., 1964, 33, 149-160. (b)

Scandura, J. M. Problem solving and prior learning. Washington, D. C.: U. S. Office of Education, 1965 (Tech. Rep. S-097). (a)

Scandura, J. M. Algorithm learning and problem solving. Washington, D. C.: U. S. Office of Education, 1965 (Tech. Rep. S-097). (b)

Footnotes

[1]This research was supported by the Cooperative Research Program of the U. S. Office of Education, Project S-097. This report, in slightly modified form, may also appear in a special research publication of the National Council of Teachers of Mathematics.

[2]Dashes, -, represent symbols used in solution sets and encircled dashes, ⊖, represent those symbols that correspond to +1 object-pair properties. The number in parentheses indicates the number of encircled dashes (i.e., symbols) in each solution set. For more details see the article by Scandura (1964a).

TABLE 1*

Presentation Orders

	L				M						
	PC		CP		PC		CP		Controls		
Section	Pr	No Pr	Pr	No Pr	Pr	No Pr	Pr	No Pr	CC	LC	MC
1	Sym	Sym	C	C	Sym	Sym	C	C	Sym	Sym	
2	Pr	Pr	Sym	Sym	Pr		Sym	Sym	Pr	Pr	
3	KR	KR	Pr	Pr	KR		Pr		KR	KR	
4	+all	+all	KR	KR	+all	+all	KR		+all	+all	
5	Pr	Pr	+all	+all	Pr		+all	+all	Pr	Pr	
6	KR	KR	Pr	Pr	KR		Pr		KR	KR	
7	+1	+1	KR	KR	+1	+1	KR		+1		
8	Pr		+1	+1	Pr		+1	+1	Pr		
9	KR		Pr		KR		Pr		KR		
10	C	C	KR		C	C	KR		C	C	C

*Sym, Pr, KR, +all, +1, C represent symbol material, practice, knowledge of results, +all material, +1 material, and criterion material, respectively. Blanks indicate the presence of filler material or corresponding practice. The filler material consisted of divisibility rules.

TABLE 2

Summary of Means (Standard Deviations)

	L				M						
	PC		CP		PC		CP		Controls		
	Pr	No Pr	Pr	No Pr	Pr	No Pr	Pr	No Pr	CC	LC	MC
R1	5.0 (2.9)	3.3 (2.7)	1.5 (2.8)	1.9 (2.6)	2.3 (2.7)	.9 (1.6)	.5 (.8)	.8 (1.8)	3.3 (2.7)	2.2 (2.5)	.3 (1.0)
R2	6.7 (2.0)	4.9 (2.6)	3.9 (3.5)	4.1 (3.6)	2.9 (3.0)	3.1 (2.5)	1.4 (1.5)	2.5 (2.9)	4.5 (2.7)	3.1 (3.2)	.7 (2.1)
R3	6.9 (1.3)	7.3 (1.1)	5.2 (3.1)	5.8 (3.2)	4.1 (3.4)	4.5 (3.5)	3.0 (3.1)	4.9 (3.3)	5.4 (2.8)	4.6 (2.7)	.4 (1.5)
R4	7.6 (1.3)	6.8 (1.7)	6.5 (2.3)	6.1 (2.5)	5.3 (2.9)	5.3 (3.2)	3.7 (2.8)	4.9 (2.7)	6.3 (2.3)	5.3 (2.5)	1.3 (2.6)
R5	7.3 (1.6)	7.0 (1.5)	5.8 (2.8)	6.2 (3.0)	3.9 (3.5)	4.7 (3.5)	2.5 (3.5)	4.2 (3.6)	6.4 (2.6)	4.8 (3.0)	1.2 (2.8)
R6	7.7 (1.0)	7.7 (.6)	6.5 (2.6)	7.3 (1.6)	5.6 (2.7)	5.6 (3.1)	5.5 (2.5)	5.9 (2.6)	6.9 (1.5)	6.9 (1.7)	2.4 (2.4)
R7	7.3 (1.4)	7.4 (1.2)	6.5 (2.8)	6.1 (3.0)	4.7 (3.7)	4.7 (3.5)	4.5 (3.3)	5.9 (3.0)	6.9 (2.6)	5.1 (2.9)	.5 (1.1)
R total	48.3 (8.6)	44.3 (8.0)	36.0 (13.5)	37.5 (15.0)	28.9 (18.7)	28.7 (17.4)	21.1 (12.6)	29.1 (14.8)	39.7 (14.3)	31.7 (15.0)	6.7 (9.9)

TABLE 2 Cont.

	L				M				Controls		
	PC		CP		PC		CP				
	Pr	No Pr	Pr	No Pr	Pr	No Pr	Pr	No Pr	CC	LC	MC
+l	8.1 (2.8)	6.6 (3.4)	8.8 (3.4)	5.4 (4.0)	6.8 (3.5)	5.7 (3.0)	6.7 (3.6)	4.0 (3.5)	10.0 (0.0)	2.9 (1.4)	4.9 (1.6)
+all	13.3 (2.0)	13.8 (.4)	13.5 (2.1)	13.9 (.4)	13.0 (2.7)	12.1 (3.0)	11.7 (4.0)	12.2 (3.8)	13.3 (2.6)	13.9 (.3)	6.8 (2.7)
Sym	6.0 (0.0)	6.0 (0.0)	5.3 (1.8)	5.8 (.8)	4.8 (2.5)	1.5 (2.6)	3.3 (3.0)	1.6 (2.7)	5.7 (1.6)	5.6 (1.5)	.3 (1.0)
N total	11.7 (6.1)	11.1 (4.7)	8.8 (4.8)	8.0 (5.9)	7.9 (7.0)	4.7 (5.3)	5.7 (5.7)	8.1 (6.4)	13.3 (5.8)	7.1 (5.1)	1.7 (1.8)
-all	11.8 (2.4)	12.4 (1.5)	12.4 (.8)	10.1 (4.5)	10.3 (4.1)	10.5 (3.8)	8.3 (4.8)	8.8 (4.5)	13.0 (.4)	10.6 (4.5)	6.5 (2.9)
C time	4.7 (3.1)	3.4 (1.2)	2.3 (1.2)	1.5 (1.1)	3.2 (.9)	3.1 (1.8)	2.9 (1.1)	2.7 (1.4)	3.5 (1.3)	4.4 (1.8)	3.3 (2.3)
R time	31.2 (9.7)	26.1 (8.2)	28.1 (8.2)	28.7 (7.0)	27.2 (7.3)	21.2 (7.8)	23.1 (8.0)	23.2 (7.3)	29.0 (7.0)	23.3 (5.5)	18.0 (5.7)
N time	7.2 (3.0)	5.9 (2.5)	6.2 (2.7)	5.3 (2.4)	5.7 (3.5)	3.6 (1.7)	6.5 (4.3)	3.9 (2.0)	4.2 (1.4)	6.3 (3.7)	3.5 (2.3)
Over-all time	81.9 (18.0)	77.9 (19.1)	81.1 (16.8)	69.9 (11.8)	81.1 (22.2)	80.5 (18.5)	75.2 (15.6)	71.5 (10.3)	70.6 (9.8)	74.0 (13.1)	73.8 (12.5)

TABLE 3

Summary of Significant F-values[1]

	R1	R2	R3	R4	R5	R6	R7	R total
Am't Prereq. (A)	18.16***	22.34***	16.35***	18.46***	25.18***	15.69***	12.60***	32.18***
Order (O)	15.61***	7.97**	3.19	4.46*	3.71			6.65*
Practice (P)								
A x O	2.91							
A x P								
O x P	4.71*	2.08						

	+1	+all	sym	N total	-all	C time	R time	N time	Over-all time
Am't Prereq. (A)	5.32*	7.78**	64.84***	10.03**	11.13**		11.01**	5.88*	
Order (O)					4.25*	17.77***			3.45
Practice (P)	12.06***		9.25**			3.75	3.27	11.27**	
A x O				2.92		9.46**			
A x P			13.46***						
O x P							4.06*		

* Significant at .05 level ** Significant at .01 level *** Significant at .001 level

[1]Blanks indicate F less than 2.75 (.10 level)

J. exp. Educ., in press

Supplement: Prerequisite Practice and Criterion Form in Mathematics Learning[1]

Joseph M. Scandura and Merlyn Behr

Florida State University

In a series of studies (1, 2, 3, 4, 5) it has been found that unless S can operationally make use of a term, statement, or rule, an explanation of a superordinate rule, using prerequisite notions, is essentially not understandable (i. e., cannot be used effectively). In particular, practice in using prerequisite notions has been shown to be one means of increasing the necessary familiarity with prerequisite material (3, 5). In all of these experiments (1, 2. 3, 4, 5), use was made of some abstract material (1) devised for use in problem solving research.

In this supplement, a related pilot study, involving actual mathematical materials, is outlined. It was hypothesized that practice in applying what appears to be prerequisite material would facilitate criterion learning only when the terms used to denote the prerequisite material consequently were used explicitly in describing the criterion material.

Method. - Basically, a 2 x 2 factorial design with repeated measures, was used. The original design, in which each S was given each treatment, included four sets of mathematical material which were originally unfamiliar to the Ss (i. e., matrix multiplication, derivative of polynomials, divisibility rules, and the game, NIM). Both materials and order of presentation were counterbalanced over treatments.

---

[1]The authors would like to thank Judith Anderson for her assistance with the data analysis.

One factor in the 2 x 2 design was prerequisite practice, given or not given. The second factor was criterion form. Two descriptions of how to solve the criterion task were prepared, one description utilizing prerequisite terminology and the other not. For example, matrix multiplication was described in terms of matrix symbolism (e. g., $[a_{ij}]$ $i,j = 1,2$) in two treatments and in more familiar language in the others. Irrespective of which criterion form was used, the prerequisite material dealt with matrix symbolism. Practice, when provided, consisted of answering questions involving this symbolism.

The Ss were 32 Florida State University juniors majoring in elementary education. They were tested during their regular class periods. One hour-period was allotted for each of the four treatments. Each treatment booklet consisted of prerequisite material with or without practice, one form of the criterion, and some test questions based solely on the criterion. They were told simply to study the material in preparation for a test. The experiment was self-paced.

Unfortunately, the basic design was contaminated when the regular instructor gave the Ss a homework assignment which was directly relevant to one set of materials (divisibility) and indirectly to another (NIM).

Results and Discussion. - General linear hypothesis methods were used to analyze the uncontaminated data. In spite of losing half of our data, the interaction between prerequisite practice and criterion form attained a suspect level ($.05 < p < .10$). The overall effect due to practice also attained the .10 level but, in view of the obtained interaction, this result is of secondary importance. Prerequisite practice improved test performance only when the criterion was described in terms of the prerequisite notions.

Mere exposure to new terminology may provide an insufficient base for describing higher order material. It may be necessary for S to be able to easily recall prerequisite notions in order for him to acquire superordinate knowledge.

Insufficient familiarity with prerequisite terminology is a real problem in many mathematics classrooms, particularly at the college level, where the material presented during single lessons is typically hierarchical in nature. Instructors should perhaps bear in mind that desired behavior may often be imparted by presenting the criterion material in a form which depends only on requisites already had by the Ss.

References

1. Scandura, J. M. "Abstract Card Tasks for use in Problem Solving Research " Journal of Experimental Education, XXXIII, No. 2 (Winter, 1964), pp. 145-48.

2. Scandura, J M. "An Analysis of Exposition and Discovery Modes of Problem Solving Instruction" Journal of Experimental Education XXXIII, No. 2 (Winter, 1964), pp. 149-160.

3. Scandura, J. M. " Problem Solving and Prior Learning" Journal of Experimental Education (1965), in press.

4. Scandura,J. M. "Algorithm Learning and Problem Solving" Submitted Journal of Experimental Education.

5. Scandura, J. M. "Prior Learning, Presentation Order, and Prerequisite Practice in Problem Solving" Submitted Journal of Experimental Education.

THE JOURNAL OF EXPERIMENTAL EDUCATION
(Volume 33, Number 2, Winter 1964)

ABSTRACT CARD TASKS FOR USE IN PROBLEM SOLVING RESEARCH

JOSEPH M. SCANDURA
State University of New York at Buffalo

THE MATERIALS used to study problem solving have varied so greatly (5) that it has been difficult to collate many of the findings. It has been especially difficult to bridge the vast chasm between problem solving research in the classroom and in the laboratory. Much of this difficulty stems from a scarcity of suitable research materials.

The purpose of this article is to present some abstract card material which has been found applicable, at most educational levels, to research in both the classroom and the laboratory (6, 7). This material was designed to meet the following criteria.

1. Prior knowledge and problem solving sophistication should have a minimal effect on learning.
2. It should be possible to construct a wide variety of related multi-stage problems.

CARD MATERIAL

The following material is related to the card tasks used in Bruner, Goodnow, and Austin's (1) study of concept attainment. Sixty-four cards, each having two objects, are used. Each object is either large or small, black or white, and a circle or a triangle. Twelve symbols are introduced: L_1, S_1, B_1, W_1, C_1, T_1, L_2, S_2, B_2, W_2, C_2, T_2. Respectively, they represent: first object is large, ..., first object is black, ..., first object is a circle, ..., second object is small, ..., second object is white, ..., second object is a triangle, ... Subsets of this set of symbols are given multiple meaning. For example, the subset, $[S_1\ C_1\ B_2]$, stands for the first object is a small circle and the second object if black.

Relationships between subsets and a card are readily specified, by attaching to the card one of the labels; +all, -all, +1, -1, ..., -6. The label +all was used to specify the relationship "all symbols in a subset agree with the card-properties." The other labels, respectively, specified; none of the symbols agree, exactly one symbol agrees, exactly one does not agree, ..., exactly six do not agree.

By labeling a card in this manner, it is possible to determine whether a given subset is associated (i.e., has the specified relationship) with it. Before going on, the reader should satisfy himself that the subsets to the right of each labeled card in Figure 1 are associated with the card in the manner indicated.

Since a subset may "go" with several cards simultaneously, it is possible to construct multi-card problems in which S is required to find every symbol subset that is associated with each card[1]. These multi-card problems can be made more or less difficult according to the number of cards and the type of labels used. Problems which include only +all cards typically have given subjects the least difficulty. It has been found convenient (in order to limit the range of difficulty) to consider only problems which include at least one +all or one -all card.

There is a logical procedure for solving each problem. Consider the problem shown in Figure 2. The first +all card establishes logically that every symbol of each subset associated with this card must be one of: S_1, B_1, C_1, L_2, W_2, T_2. That is, each associated subset can contain only elements chosen from among these six symbols. The second +all card does not have any of the properties represented by the symbols B_1, C_1, W_2, T_2. Thus, each nonempty subset which goes with both of the +all cards must contain one or more of the remaining symbols, in this case, S_1 and L_2. The only subsets that go with both of the +all cards are $[S_1]$, $[L_2]$, and $[S_1 L_2]$. No other subset will do. The subset $[S_1 W_2]$, for instance, although going with the first +all card does not go with the second.

Since the +1 card has both of the properties indicated by the two eligible symbols, S_1 and L_2, the one-symbol subsets $[S_1]$ and $[L_2]$ are the only ones which go with the +1 card as well as with the two

+all cards. The subset [S_1 L_2], for example, although it goes with the two +all cards, does not go with the +1 card. By definition, exactly one of the symbols must agree; instead, both S_1 and L_2 agree.

Procedures for solving any multi-card problem may be obtained by a similar sort of analysis. Two more difficult problems are presented, with solutions, in Figure 3. The reader may find it instructive to solve the problems himself before looking at the solutions. Other variations are readily constructed.

APPLICATIONS

To date, the card material has been used solely to explore selected aspects of the teaching-learning process (6, 7). Readiness, transfer, directness and meaning were the primary variables of concern. The manner in which the material was used in two of these studies is described below.

One study involved a comparison of two often used instructional methods, one called exposition (E) and the other discovery (D). In both classrooms, a large board, displaying all 64 cards, was in sight – this was found useful in introducing the material. Colored chalk was used to construct card problems on the blackboard[2].

In the E-classroom, the experimenter simply stated the card properties along with the symbol and subset meanings. Then, the +all relationship, between subsets and cards, was explained. This was done briefly and no student participation was elicited.

Next, the problem solving algorithm was shown and the steps made clear – little, if any, emphasis was given to the underlying rationale. Ss spent the remaining seventy percent of the class time (70 percent of 67 minutes) solving two and three +all card-problems. A competitive situation, pitting boys versus girls, was produced and the "practice" took place under conditions of high motivation.

In the D classroom, the experimenter did not tell the Ss anything directly. Questions were asked and hints were given which were designed to challenge and to aid the Ss to find systematic algorithms for solving the card problems. "How do these cards differ? S_1 means the first object is small; what do you suppose L_1 means? Has anybody figured out what makes a 'word' go with a +all card? Can anyone find a 'three-letter word' going with this (+all) card? Can anyone tell me how many associated two-letter words there are that go with these three +all cards?" These were typical questions; the purpose of the last was to induce the Ss to find systematic modes of attack. After the Ss generally had attained a good problem solving facility, the experimenter changed topics. He wrote six symbols on the board and asked the Ss to write all of the possible subsets containing these symbols. After the Ss realized how tedious the task was (there are 63 non-empty possibilities), the task was made simpler by crossing off one or more symbols. This technique was repeated until the Ss generally were able to attain every subset in an orderly fashion. When the Ss were asked, "Can anyone solve this problem by 'crossing out' symbols?" several of the best Ss caught on almost immediately – the idea spread rapidly. One more problem was given and the instruction concluded after 67 minutes.

Posttesting involved two short tests given after a short refreshment break. The first, a routine test, was composed of +all problems and was used as a measure of specific transfer. The second, a novel test, contained problems made up of both +all and -all cards and was used as a measure of non-specific transfer[3].

As is typical in classroom research, several variables were confounded. Nonetheless, by a careful post-analysis of tape-recordings of the classes, valuable insights into the interrelationships involved were obtained. These are reported elsewhere (6, 7) and there is no need to repeat them here.

A similar study was conducted in the laboratory. Two groups of Ss, pretimed (P) and timed (T), were trained and tested individually. The nature of the cards, symbols, and the relationships between them were described in the same manner to all Ss. Group P Ss were shown the problem solving algorithm before having had an opportunity to attack a series of 10 problems while group T received the information afterwards. Both groups were tested for specific and non-specific transfer. Group P was slightly superior with regard to specific transfer, but group T transfered 50 percent more in the non-specific situation.

In other explorations, the card material has been taught for as long as five lessons of an hour each at grade levels ranging from the fourth to a college calculus class. Instructional time has been found to vary as a function of problem complexity, the kind of instruction given and the problem solving sophistication of the Ss.

DISCUSSION

It is felt that the card material has important advantages over ordinary subject matter as a tool in educational research.

1. By serving to equate subject-pre-familiarity; thus, eliminating the need for pre- and post-testing; fundamental changes in behavior may be more readily detected.
2. The sequential nature of the card material and the many kinds of related problems that can be constructed from it make the material particularly well suited for use in research involving readiness and transfer. Both variables are fundamental to the teaching-learning process, but have always been somewhat elusive.

3. The card problems are similar to many problems confronted in the classroom, particularly the mathematics classroom. Thus, findings based on card-material-research may be more directly generalized to the classroom than have many laboratory results. By "simulating" the classroom learning of ordinary subject matter with novel material some educational problems may be explored with a relatively small expenditure of time and energy. Such an approach may make it possible to systematically collect a body of experimental data in much the same manner as has been done in the study of learning.

Furthermore, since the material appears to be applicable to the classroom and laboratory alike, researchers who are interested in both levels of investigation have a tool which may aid in the coordination of their work. Educational research may raise questions for more precise study in the laboratory and laboratory findings may be tested for application in the classroom in a more integrated manner than has been the case.

Although the card material, to date, has been exclusively employed to study the teaching-learning process, it shows promise in other areas of problem solving research. Application to studies involving problem complexity (2) and comparisons of problem solving ability at various age and ability levels (3) appear promising.

SUMMARY

Some abstract card material was described and its use illustrated in both educational and laboratory situations. Advantages in using the card material to simulate classroom situations and to aid in the coordination of educational and laboratory research were outlined.

FOOTNOTES

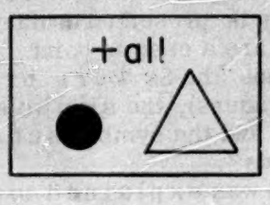
1. The collection of symbol subsets which satisfy this condition (i.e., of being associated with each card) may be called the "solution set".
2. The symbols denoting color were changed appropriately. The essential features of the card material may be presented in many ways. Maag's (4) solids are a case in point. For pedagogical reasons (the Ss were bright fourth and fifth graders), the symbols were called "letters", and the symbol subsets were called "words".
3. The -all relationship was explained to both groups in the same way just before the novel test.

REFERENCES

1. Bruner, J. S., and Goodnow, J. J., and Austin, G. A. A Study of Thinking (New York: Wiley and Sons, 1956).
2. John E. R. "Contributions to the Study of the Problem Solving Process", Psychological Monographs LXXI (1957), (Whole No. 447).
3. Kendler, H. H. and Kendler, Tracy S. "Vertical and Horizontal Processes in Problem Solving", Psychological Review LXIX (1962), pp. 1-16.
4. Maag, C. H. "Development and Evaluation of a Conceptual Reasoning Test", Educational and Psychological Measurement XVII (1957), pp. 230-39.
5. Ray, W. S. "Complex Tasks for Use in Human Problem Solving Research", Psychological Bulletin LII (1955), pp. 134-49.
6. Scandura, J. M. The Teaching-Learning Process: Exploratory Comparisons of Exposition and Discovery Modes of Problem Solving Instruction, unpublished doctoral dissertation, Syracuse University, 1962.
7. Scandura, J. M. "An Analysis of Exposition and Discovery Modes of Problem Solving Instruction", The Journal of Experimental Education XXXIII, No. 2 (Winter 1964), pp. 149-56.

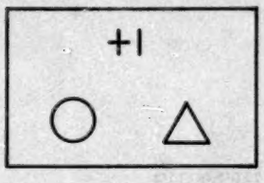
FIGURE 1

+ALL, +1, AND -3 CARDS AND TWO ASSOCIATED SUBSETS FOR EACH



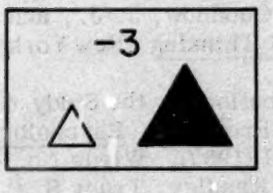
$\{S_1\}$

$\{B_1 L_2 W_2 T_2\}$



$\{S_1\}$

$\{L_1 W_2 C_2\}$

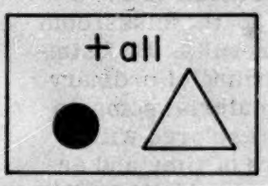


$\{L_1 W_2 C_2\}$

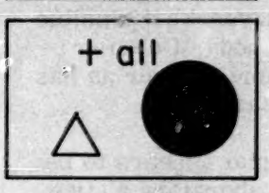
$\{W_1 C_1 L_2 W_2 C_2\}$

FIGURE 2

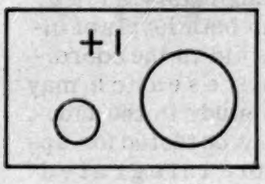
THREE-CARD PROBLEM, WITH SOLUTION SUBSETS AND PROBLEM SOLVING ALGORITHM SHOWN IN STAGES



$S_1 B_1 C_1 L_2 W_2 T_2$



$S_1 B_1 C_1 L_2 W_2 T_2$



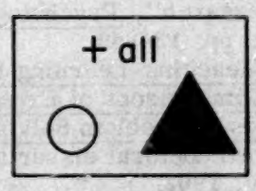
1

$S_1 L_2$

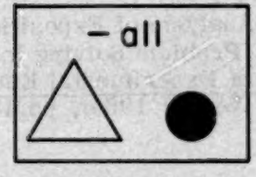
$\{S_1\}$ $\{L_2\}$

FIGURE 3

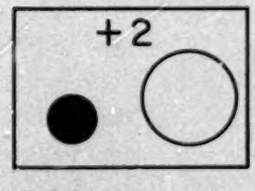
TWO THREE-CARD PROBLEMS, WITH SOLUTION SUBSETS AND PROBLEM SOLVING ALGORITHMS SHOWN IN STAGES



$S_1 W_1 C_1 L_2 B_2 T_2$



$S_1 W_1 C_1 L_2 B_2 T_2$

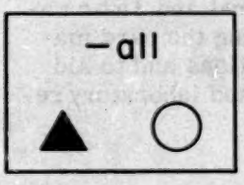


2

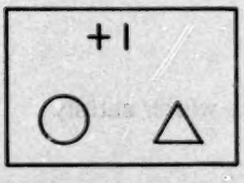
$S_1 C_1 L_2 T_2$

$\{S_1 C_1\}$ $\{S_1 L_2\}$ $\{C_1 L_2\}$

$\{S_1 C_1 T_2\}$ $\{S_1 L_2 T_2\}$ $\{C_1 L_2 T_2\}$

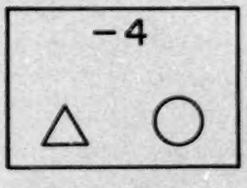


$L_1 W_1 C_1 L_2 B_2 T_2$



1

$L_1 W_1 C_1 L_2 B_2 T_2$



1

$L_1 W_1 C_1 L_2 B_2 T_2$

4

$\{C_1 L_1 L_2 B_2\}$

$\{T_2 L_1 L_2 B_2\}$

PROBLEM SOLVING

AND

PRIOR LEARNING

Joseph M. Scandura

Florida State University on behalf of
The Research Foundation of
State University of New York

Proj. No. S-097

May 1, 1964 to January 31, 1966

BACKGROUND

Good teachers have long observed that when information is given is just as important as what information is given. The implications of this observation are of real concern to all educators, particularly teachers of mathematics and science. Nonetheless, most specialists in education would be hard pressed to explain the mechanisms involved. A better understanding of these mechanisms is imperative if present developments in instructional technology are to reach their full potential. Still, the number of controlled studies which have dealt with this problem is small. Those which involve reading extended passages of technical materials have been almost nonexistent; yet, this is precisely the way in which most students learn such materials at the high school and college levels dealt with in this study. The purpose of this project was to explore various ways in which prior (prerequisite) learning might affect the learning of higher order (criterion) material which builds on this foundation. Major emphasis was given to identifying important variables and assessing their relative strengths. Towards this end, three major experiments and one supplement (over and above the contract terms) were conducted.

OBJECTIVES

The purpose of Experiment One was to determine the effects of practice at the prerequisite and criterion levels on the learning, transfer, and retention of some criterion material, as evidenced by the solving of problems defined at the criterion level. Criterion practice involved the same type of problems used to assess learning. It was hypothesized that

unless the prerequisite material was sufficiently well practiced, the learning of higher order material, whose description involved prerequisite terminology, would be relatively poor. Since the tests for transfer and retention provided only indirect measures of criterion learning, the effects of the independent variables were expected to be less on these measures.

Experiment Two was designed to determine whether information about problems is necessarily reflected in problem solving performance. It was hypothesized that prerequisite information facilitates algorithm learning only when the information is specifically used to describe the algorithm. Other information about problems reflects itself only to the extent that the algorithm, as a method, is inadequate to solve transfer problems and to the extent that the information provides a basis for additional positive transfer to such problems.

The most encompassing experiment (Three) in this series was concerned with the effects of, and interactions between, three independent variables: (1) amount of prerequisite learning prior to instruction, (2) order of presenting prerequisite and criterion materials, and (3) prerequisite practice. In this study learning was assessed at various levels of familiarity with the criterion materials and problems. It was hypothesized that: (1) criterion learning, as judged by problem solving performance, can be manipulated by prerequisite learning--only a non-causal relationship has been demonstrated in previous studies, (2) prerequisite learning facilitates problem solving by increasing the degree of criterion learning and not by facilitating problem solving directly--this hypothesis was tested by varying the order of

presenting prerequisite and criterion materials, and (3) prerequisite practice facilitates criterion learning only when the prerequisite material comes before the criterion.

The supplementary experiment was conducted to determine the feasibility of extending the results of the first three experiments to actual subject matters like mathematics. Two variables were manipulated, criterion form and prerequisite practice. The criterion was described using either the prerequisite terminology or already familiar terms. In view of the earlier results, an interaction was predicted between prerequisite practice and criterion form. Practice was assumed to facilitate criterion learning only when the criterion description involved use of the prerequisite terms.

PROCEDURE

In the first three experiments, some artificial material, constructed and published earlier by the author, was used to minimize the contaminating influence of uncontrolled individual differences in related knowledge. Four mathematical topics, unfamiliar to the students involved in these experiments, were used in the supplementary fourth experiment. In each of the experimental and control treatments, one part of the experimental material was designated criterion and the rest, prerequisite. Both the prerequisite and criterion materials were presented in paragraph form. The words, symbols, and/or graphic materials used to denote prerequisite notions were also used in presenting the criterion material (except as specified below). Prerequisites, which were logically determined, refer to those statements of principle which apparently need to be understood before new material, whose description

involves reference to these prerequisite principles, can be mastered. Operationally, understanding meant that the principles could be applied. In the first and third experiments the criterion material consisted of a definition of the training problems, an illustrative problem and its solution, and pointing out relationships between the problem and its solution. In the second experiment, the criterion material consisted essentially of an efficient algorithm for solving the training problems. Each criterion topic, used in the fourth study, consisted of a description of a simple mathematical concept, one of which was matrix multiplication. Learning, transfer, and retention test problems were constructed for use in the first two studies. The third experiment involved only learning and transfer problems, while the fourth involved a test based directly on the criterion material.

The subjects used in the first two experiments were high school juniors taking college preparatory mathematics, 80 in Experiment One and 84 in Experiment Two. The third and fourth experiments were conducted with college students enrolled in a mathematics education course for elementary teachers. There were 165 students in Experiment Three and 32 in Experiment Four. Participation was a class requirement in each case.

The first two experiments were conducted in parallel fashion, each dealing with a separate aspect of the overall problem. In the first experiment, entitled "Problem Solving and Prior Learning," two variables were manipulated independently, practice in applying prerequisite material and practice in solving the problems defined at the criterion level. In each case, feedback was given.

The experiment entitled "Algorithm Learning and Problem Solving " was

concerned with the amount of information about problems and its effects on the ability to use and generalize an algorithm for solving similar problems. There were four levels of this variable: no prerequisite material at all, only that part of the prerequisite material deemed necessary for learning the algorithm, information deemed necessary for promoting transfer and, finally, definition of the problems themselves (the criterion in Experiment One).

The third experiment, entitled "Prior Learning, Presentation Order, and Prerequisite Practice in Problem Solving," involved a 2 x 2 x 2 factorial design along with three control treatments. The variables were: (1) amount of prerequisite training (to a high learning criterion) prior to actual experimentation, (2) order of presentation of the prerequisite and criterion materials, and (3) prerequisite practice. In one control, none of the prerequisite material was learned either before or during the experiment. In another, some of the prerequisite material was learned just prior to the experiment; the rest of the prerequisite material was not presented during the experiment (this control group corresponded to those experimental groups receiving prerequisite training prior to the experiment). The third control group prelearned all of the prerequisite material prior to the experiment. All of the experimental subjects were tested on the training problems after the criterion had been presented once and six times thereafter, interspersed with repeated presentations of the criterion, additional hints, and test problem feedback.

The fourth experiment, entitled "Prerequisite Practice and Criterion Form in Mathematics Learning," involved basically a 2 x 2 factorial design

with repeated measures. The original design, in which each experimental subject received each treatment, involved four mathematical topics which were originally unfamiliar to the subjects (i.e., matrix multiplication, derivative of polynomials, divisibility rules, and the game NIM). Both materials and order of presentation were counterbalanced over treatments. Unfortunately, the basic design was contaminated when the regular instructor gave the subjects (his students) a homework assignment which was directly relevant to one set of materials (divisibility) and indirectly to another (NIM).

Each experiment was run in groups of between 15 and 40 subjects. The subjects were simply instructed to study the material as they would be tested on it. The first two experiments were experimenter-paced. The third and fourth were self-paced since holding time constant, in Experiments One and Two, appeared to serve no useful purpose and since self-pacing frequently reduces uncontrolled variance. In the third experiment, involving pretraining, all subjects reported at the same time. Those subjects receiving pretraining were taught in one room while the others discussed some unrelated mathematics problems in a second classroom. After pretraining, all of the subjects were brought together in a single room for the experiment. Where possible, directions were given by tape recorder for uniformity.

The primary dependent variable in these experiments was the percentage of correct or partially correct solutions given to the test problems. In addition, learning and test time, along with the amount of work shown on the papers, was recorded, where appropriate. These latter measures were not particularly helpful, however, since they probably reflected both learning and motivation (persistence) factors.

Standard analysis of variance procedures provided the basis for most of the inferences. Nonetheless, other measures and techniques were used where they seemed appropriate. For example, correlational measures were used to estimate test reliabilities (which were about .90). In addition, the contamination problem noted in Experiment Four made it necessary to use more general linear regression methods to analyze the data.

RESULTS

The results of Experiments One and Two were essentially as predicted. Practice at the prerequisite level, with feedback, facilitated problem solving performance whereas criterion practice did not (Experiment One). The corresponding effects on the transfer tests, however, were smaller. In Experiment Two, exposure to information about problems reliably improved problem solving performance, via an efficient algorithm, but only when the information was either specifically used to describe the algorithm or clearly provided a basis for modifying the algorithm taught so as to make solution of the transfer problems possible. Other information about the problems, and particularly definition of the problems, did not facilitate problem solving in any case. The results of Experiment Three were also as predicted: (1) prelearning the prerequisite material had a highly reliable facilitating effect on problem solving, (2) problem solving performance was better when the prerequisite material was presented first and the criterion second than when the order was reversed--the effect disappeared after repeated reintroduction of the criterion, and (3) prerequisite practice improved problem solving only when the prerequisite material

came before the criterion--the practice effect was relatively weak and transitory. Finally, although only half of the Experiment Four data was usable, the results were in the predicted direction (at the .10 level). Prerequisite practice facilitated criterion learning only when prerequisite terminology was used in describing the mathematical criteria.

Perhaps the major result of the entire project, however, was unanticipated. This concerned the long-lasting effect of prerequisite prelearning on criterion learning in Experiment Three. The repeated reintroduction of the criterion, along with additional hints and practice in solving the training problems with feedback, failed to diminish the superiority originally evident with the pretrained groups.

CONCLUSIONS AND IMPLICATIONS

Although there were a variety of peripheral results, four fundamental conclusions may be drawn from this project.

1. Practice in applying prerequisite terminology significantly improves the learning of higher order material, as judged by criterion test performance, when the description of the criterion material involves prerequisite terminology (Experiments One and Four). This conclusion implies that teachers, particularly of mathematics and probably of science, should be careful to make provision for practice with newly introduced prerequisite terms before using these terms to describe higher order materials. This is not always done in high school and college classrooms.

2. The effects of prerequisite practice obtain only when the criterion description involves use of prerequisite terminology (Experiments Two and Four).

In effect, there are often alternative means available for achieving a desired objective and these should be considered in those cases where the teacher is primarily interested in promoting acceptable criterion performance with the least expenditure of time. This possibility was demonstrated in Experiment Two by the use of an efficient algorithm and in Experiment Four by the use of alternative criterion descriptions in familiar, as opposed to prerequisite, terms. An incidental implication of this conclusion, rather glaringly evident in Experiment Two, is that it is possible to solve problems and even generalize solution procedures to new problems without even knowing what the problem is. This finding suggests that understanding is an imprecise term. The educationally relevant question is not understanding versus no understanding, but simply what is learned.

3. Learning prerequisite material, via practice or otherwise, does not affect criterion test performance directly, but indirectly by increasing criterion learning. This conclusion may be drawn from the Experiment Three result in which the order prerequisite-criterion was superior to the order criterion-prerequisite. The interaction between prerequisite practice and presentation order (Experiment Three) provides further support. This conclusion makes it possible to discard the possibility that prerequisite learning simply amounts to learning part of the task or that a substantial amount of criterion material can be retained and then correctly interpreted when the prerequisite meanings later become available. Insofar as possible, prerequisites should be taught first and not after the criterion to insure efficient learning.

4. The most far-reaching conclusion to be drawn from this project is that learning ability is far from innate; it depends fundamentally on the

amount of prerequisite learning already available. Furthermore, it may be concluded that spending more time on criteria and having an opportunity to practice on related problems, cannot be expected to diminish the advantage of prior prerequisite knowledge. Presumably, the only way this can be done is by going back and teaching the prerequisites to the lacking students.

The practical implications of this conclusion seem clear. Just because a student from one of our top high schools, for example, does a better job in college calculus than a student from an underprivileged background, is no reason to believe that this same outcome would obtain if the underprivileged student had had the opportunity of equivalent prior training. Since the effects of such pretraining are probably cumulative, we need to devise ways of salvaging those students who would normally fall by the wayside, probably by individualizing instruction. Although developmental activities are needed now, it should be emphasized that we know precious little about the fundamental bases on which individualized instruction rests. Clearly, more basic research on this problem is needed.

BIBLIOGRAPHY

There are 29 different references listed in the final report.

PUBLICATIONS

All four experiments, conducted during the course of this project, have been accepted for publication in the Journal of Experimental Education. They will appear simultaneously, some time during 1966, under the titles:

(1) Problem Solving and Prior Learning,

(2) Algorithm Learning and Problem Solving,

(3) Prior Learning, Presentation Order, and Prerequisite Practice in Problem Solving, and

(4) Supplement: Prerequisite Practice and Criterion Form in Mathematics Learning.